CS 366
Assignment #1
Matrix Multiplication Matters
Due 1/24/18, in class

1. Goal
The goal of this assignment is to better understand three variations of matrix multiplication.

2. Problem Statement [client’s statement of their need]
Mr. Krabs needs a lightning quick matrix multiplier.

Back Story: Mr. Krabs has three suppliers for crab, lettuce, buns, sauce, etc. Each charges different amounts for each ingredient. He also knows how much of each he uses each day. Sponge Bob told him that to compute the cost for each ingredient on each day he need only do some matrix multiplication. Mr. Krabs has hired you (and not the sponge) to implement this matrix multiplication.

3. Analysis [What is the client’s problem?] Given a problem statement, an analyst asks questions to better understand what exactly the client’s problem is (clients are notorious for not knowing their own problems!). In this assignment there are three key questions “What is a matrix?”, “What are the steps one takes to multiply two matrices?”, and “What is the process to select each day’s supplier?”

To answer the first, a programmer thinks of a matrix as a two-dimensional array of numbers. For the second, to multiply two matrices involves computing the dot product of various rows and columns. Rather than repeat it here, the following web page describes the process in detail http://mathworld.wolfram.com/MatrixMultiplication.html, while https://www.mathsisfun.com/algebra/matrix-multiplying.html provides a simple example.

In our case, Mr. Krabs purchased an algorithm for you to implement (from Plankton’s algorithm emporium). Call this Version 1. In this algorithm the values in the product matrix are computed one by one using a loop that iterates \( k \) times to compute the dot product of row \( r \) from the first matrix and column \( c \) from the second. In pseudo code this is

\[
\text{for each row } r \\
\quad \text{for each column } c \\
\quad \quad \text{for each index } k \\
\quad \quad \quad \text{product}[r][c] += A[r,k] \times B[k,c]
\]

assuming that \( \text{product} \) is initialized to all zeros.

Filling the matrices Mr. Krabs will use into the algorithm above, yields

\[
\text{for each row } r = \text{Monday then Tuesday} \\
\quad \text{for each column } c = \text{Sandy then Squidward} \\
\quad \quad \text{for each index } k = 1 \text{ then } 2 \\
\quad \quad \quad \text{product}[r, c] += \text{usage}[r, k] \times \text{costs}[k, c]
\]

The final question is best answered with an example. Consider the following product.

<table>
<thead>
<tr>
<th>cost matrix</th>
<th>usage matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crab Lettuce</td>
<td>Sandy’s Supplies</td>
</tr>
<tr>
<td>Monday 8 1</td>
<td>Crab 1.99</td>
</tr>
<tr>
<td>Tuesday 2 9</td>
<td>Lettuce 2.99</td>
</tr>
</tbody>
</table>
Here is a trace of each time product is updated

\[
\begin{align*}
product[\text{Monday}][\text{Sandy}] & += 8 \times 1.99 & \text{ // } k = 1 \\
product[\text{Monday}][\text{Sandy}] & += 1 \times 2.99 & \text{ // } k = 2 \\
product[\text{Monday}][\text{Squidward}] & += 8 \times 1.50 & \text{ // } k = 1 \\
product[\text{Monday}][\text{Squidward}] & += 1 \times 4.50 & \text{ // } k = 2 \\
product[\text{Tuesday}][\text{Sandy}] & += 2 \times 1.99 & \text{ // } k = 1 \\
product[\text{Tuesday}][\text{Sandy}] & += 9 \times 2.99 & \text{ // } k = 2 \\
product[\text{Tuesday}][\text{Squidward}] & += 2 \times 1.50 & \text{ // } k = 1 \\
product[\text{Tuesday}][\text{Squidward}] & += 9 \times 4.50 & \text{ // } k = 2
\end{align*}
\]

The resulting matrix is

\[
\begin{array}{cc}
\text{ Sandy’s } & \text{ Squidward’s } \\
\text{Monday} & 18.91 & 16.50 \\
\text{Tuesday} & 30.89 & 43.50
\end{array}
\]

Here the product matrix says that Mr. Crabs should order from Squidward on Monday (because the total cost of 16.50 is less than Sandy’s 18.91), while on Tuesday he should order from Sandy.

So far so good, but, alas, as these things go, there was what has become known in Bikini Bottom as “the debate.” Patrick said that he knows a better way — change the order of the outer two loops (this is Version 2). Sponge Bob said Patrick is wrong, you want to change the order of the inner two loops (this is Version 3). Mr. Krabs threw up his claws and has since decided to task you (he pays well) with understanding how all three work.

Assignment Requirements

1. Add Buns to the list of supplies Mr. Crabs needs daily, and also add the day Wednesday. He needs 10 Buns on Monday, 6 on Tuesday, and 3 on Wednesday. On Wednesday his needs also include 3 Crab and 4 Lettuce. Sandy charges 0.50 for Buns while Squidward charges 0.60.

2. Trace the computation of the product as done above using all three versions of the multiplication algorithm (original, Patrick’s, and Sponge Bob’s).

3. Carefully record each trace in a simple text file that begins with the header

\[
// \text{ This is my work} \\
// <\text{Your Name}> \\
// \text{CS366}
\]

Then label each of the three traces with one of the three names and the pseudo code used to produce it. At the end of all three traces weigh in on the debate by explaining which algorithm is better and why you think so.

4. Finally, for each of the three traces, use the result to answer the following question “On each day, who should Mr. Crabs purchase supplies from?”

What to hand in

1. A well-formatted 2-up printout of all three traces. You must use the linux command a2ps for this. Consider creating an a2ps alias in your bash_profile to include the following options a2ps

\[
-T 4 -q -Avirtual -2 -o my_answer.ps
\]

You can use the command gv to preview your submission and the command lpr to print it.
(2) A no-more-than one page summary of the following article (available off the course web page):

Grading

Here is a list things that I will look for
• use of a2ps,
• proper headers,
• correctness of the traces,
• correctness of who Mr. Crabs should purchase supplies from, and
• insight in your summary.