Model Projection: Simplifying Models in Response to Restricting the Environment

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ABSTRACT

This paper introduces Model Projection. Finite state models such as Extended Finite State Machines are being used in an ever increasing number of software engineering activities. Model projection facilitates model development by specializing models for a specific operating environment. A projection is useful in many design-level applications including specification reuse and property verification.

The applicability of model projection rests upon three critical concerns: correctness, effectiveness, and efficiency, all of which are addressed in this paper. We introduce four related algorithms for model projection and prove each correct. We also present an empirical underpinning of model projection as formalized in Section 2. Section 3 then introduces four progressively more precise and eight transitions. This substantial reduction in controller size requires consideration of a state space having over 3,000 states. How-ever, only four events affect the restricted environment. This causes the state Sugar to become unreachable and subsequently the state Double Sugar, because it is only reachable from the state Sugar. When these states and their associated transitions are projected out of the model, only the black elements remain.

1. INTRODUCTION

Both automatically and manually generated state-based models find many applications in software engineering. For example, they are often constructed by model checkers [8] and built by engineers in the telecommunications and embedded systems sectors [9, 10, 25]. They are also used in modelling notations such as the UML [28], which draws heavily on state-based modelling [27], and are widely used for specifying discrete-event control devices, such as automated manufacturing systems (AMS) [30].

A great deal of engineering effort is directed towards the design of an organisation’s models. As a result, it is common for models to be reused. One such scenario arises when control models are reused within a device that offers a restricted operational environment. For example, the ‘basic model’ car air conditioning system may not offer the climate control functionality found in more luxurious models. It would be wasteful to specify different models for each potential car configuration. Rather, there is typically a single controller for the entire product line [3]. In this situation, model projections, specialized to the particular environment, are useful as they avoid producing bloated controllers that implement unused functionality.

The design of models typically occurs earlier in the overall development process than coding where the cost of errors is, in general, higher. This makes it important to have powerful and flexible tools for model analysis. Model projection facilitates such analysis in much the same way that techniques such as program transformation and program slicing assist with code level analysis [29, 16, 32].

Figure 1 presents an example of model projection in which a vending machine allows a user to insert coins and vends tea with options of large cups, small cups, sugar, and double sugar. Consider redeploying this controller in an environment where the ‘sugar’ button is disabled, and thus in an environment in which the event Sugar() never occurs. The projection removes all states and transitions (shown in dashed grey) unreachable by event sequences in the restricted environment. This causes the state Sugar to become unreachable and subsequently the state Double Sugar, because it is only reachable from the state Sugar. When these states and their associated transitions are projected out of the model, only the black elements remain.

Model projection can also be used to facilitate property verification by reducing the complexity of a model and thus making analysis more efficient. Consider the production cell example used in the industrial case study of a German metal processing plant [26]. In this system, metal blanks enter the system on a feed belt and are conveyed via a table by robot arms to one of two presses. Assume that we want to prove that the feed belt motor is off if there is a blank on the belt and a blank on the elevating rotating table. Call this property P. It turns out that property P only concerns two sensors and one actuator of the model. To prove that P holds in the original model requires consideration of a state space having over 3,000 states. However, only four events affect P (s1on, s1off, s3on, s3off). A model projection produced by ignoring all other events has only four states and eight transitions. This substantial reduction in controller size means that P can be verified to hold in all states directly. As shown in Section 4, this implies that P holds in the original model.

With these applications and others, the applicability of our approach depends upon both correctness and the reduction achieved in reasonable time. Therefore, this paper concerns the theoretical and empirical underpinnings of model projection as formalized in Section 2. Section 3 then introduces four progressively more precise (and more expensive) model projection algorithms. This is followed in Section 4 by proofs of correctness for the four algorithms. Finally, Section 5 investigates the effectiveness and efficiency of our approach in terms of the reduction achieved on a set of ten models (including standard benchmarks and real world models).
2. MODEL PROJECTION DEFINITION

This section first introduces some notation and terminology related to the particular models focused on in this paper: Extended Finite State Machines (EFSMs). It then formalizes the notion of a projection for EFSMs.

Definition 1 (Extended Finite State Machines (EFSMs))
An EFSM $M$ is a tuple $(S, T, A, S_i)$ where $S$ is a set of states, $T$ is a set of state to state transitions, $A$ is an event alphabet (a set of events), and $S_i$ is the set of initial states. A transition $t \in T$ has a label $lbl(t)$ of the form $e(g)/a$, where $e \in A$, $g$ is a boolean guard, written in an unspecified condition language, and $a$ is an action, written in an unspecified command language. All parts of the label are optional. Only when the guard evaluates to True can the transition be taken. An omitted guard is assumed to be True. For transition $t$, source($t$), target($t$), guard($t$), trigger($t$), and action($t$) are used to refer to $t$’s source state, target state, boolean guard, trigger event, and action, respectively.

An EFSM $M$ takes as input a sequence of events $E \in A^*$. Each event includes, in addition to the event name, a list of formal parameters that are passed values from the external environment with the event. A transition’s action commonly updates the store and may be used to refer to

Definition 2 (Implicit Transition Free Event Sequence)
For EFSM $M = (S, T, A, S_i)$, an input event sequence $E \in A^*$ is referred to as implicit transition free if $M$ takes no implicit transitions when presented with $E$. Such an event sequence is also referred to as stutter-free. For a set of event sequences $X$, $\text{if}(M, X)$ denotes the implicit transition free subset of $X$:

\[ \text{if}(M, X) = \{ E \in X \mid M \text{ takes no implicit transitions when processing } E \}. \]

Figure 1: Illustrative example: model projection for a simple vending machine. The complete vending machine includes both the dashed grey and black elements. The projection, for reuse in an environment where sugar is unavailable (the ‘sugar’ button is disabled), includes only the black elements of the model. The states and transitions concerned with providing sugar are projected out of the model. The projection is faithful to all other (sugar free) interactions with its environment.

Figure 2: The basic model projection.

Given a set of events to ignore, denoted $I$, the projection of EFSM $M$ preserves $M$’s behavior when placed in an environment that does not generate events from $I$. The resulting set of event sequences, formalized by the following filter, is used to define model projection.

Definition 3 (Event Sequence Set Filter)
Sequence-set Filter function $\text{fi}(X, I) = \{ E \in X \mid E \cap I = \emptyset \}$ where $E \cap I$ denotes the intersection of the set of events in $E$ with $I$.

Definition 4 (Model Projection)
A Model Projection of EFSM $M = (S, T, A, S_i)$ for ignore set $I$ is a reduced EFSM $M'$ that is semantically indistinguishable from $M$ on all event sequences from $\text{fi}(A^*, I)$.

For a projection to be semantically indistinguishable as required by the definition, it must preserve the behavior of the original machine. There are two aspects to this semantic requirement: what behavior is preserved and over what set of event sequences it is preserved. The first of these, “what behavior is preserved” is formalized by Definition 9 in Section 4. Informally, the behavior includes the variable values computed at each state.

The second aspect of the semantic requirement considers the set of event sequences over which the semantics is preserved. Two options are considered: the strong semantic requirement and the weak semantic requirement. Under the strong requirement, behavior is preserved on all sequences in $\text{fi}(A^*, I)$, while for the weak requirement, behavior is preserved only for the stutter free sequences of $\text{fi}(A^*, I)$. Thus the weak requirement does not require matching behavior on event sequences for which the EFSM implicitly consumes an event. Rather, it restricts this requirement to event sequences where the behavior of $M$ is explicitly defined.

Figure 2 presents the definition of a basic projection of an EFSM, which is illustrated by the example in Figure 1. In Section 4 we
prove that the implementation of basicProjection shown in Figure 3 satisfies the strong semantic requirement. Furthermore this algorithms removal of states and transitions can facilitate further simplifications. Thus three additional algorithms go beyond the basic projection and consider simplifying the state machine using constant propagation and state merging. The most aggressive of the state merging algorithms satisfies only the weak semantic requirement.

3. ALGORITHMS

This section describes four model projection algorithms that represent differing performance/preference tradeoffs. Model projection has several possible implementation strategies. For example, one could build a projection by adding necessary elements to an initially empty model. Alternatively, one could remove elements from the model being projected. The initial implementation follows the later approach. To do so, it recasts the notion of program slicing [34] to the EFSM domain. However, this recasting differs from previous work on slicing state-based models [17, 21, 23, 24, 33], which transplant concepts from the program slicing [34] in an attempt to incrementally adapt program-slicing concepts and techniques. These differences are considered further in Sections 5.3 and 6.

For much of its history, dependence analysis has typically been applied to programs, but there has been a recent increase in the theory and practice of dependence analysis for models, and in particular state-based models [1]. Indeed, it has been argued [15] that all descriptions, whether they be specifications, models, or other descriptions, will have a tendency to become executable over time, making such descriptions more likely to become the subject of techniques such as slicing, which is one of the most popular techniques for detecting functional equivalences in programs [34].

Figure 3: Algorithm A1: basicSlice. Basic algorithm for computing an environment-based slice.

Figure 4: Algorithm A2: constantPropagation. Basic algorithm for computing a variable assignment.

variables that appear as formal parameters of an event labelling a transition are assigned the value \( \bot \) (non-constant) because they receive an unknown value from the environment. All other variables \( v \) are initialized to \( \top \) (uninitialized). The assignments that label the transitions are then taken into account. If the right-hand side of the assignment evaluates to a constant then the value of the left-hand side variable is replaced with the meet of the constant and the variable’s present value. Otherwise, the value is replaced with \( \bot \).

In addition to constant propagation, two state merging algorithms are used to further reduce the size of a projection by combining groups of states that have identical semantics. The right-equivalent merging algorithm extends an algorithm of Ilie et al. [19] to EFSMs. R-equivalence equates to bisimulation for deterministic state
SliceMerge(EFSM M, Ignore Event Set I)
{
  M' = basicSlice(M, I)
  constantPropagation(M')
  R-mergeEquivalentStates(M')
  G-mergeEquivalentStates(M')
}

R-mergeEquivalentStates(EFSM \( M = (S, T, A, S_i) \))
{
  while \( \exists \) States \( s_1, s_2 \in S \) such that \( R\)-equivalent\( (s_1, s_2) \)
  replace \( s_1 \) and \( s_2 \) with \( s \), where \( s \) has as its outgoing (and self-) transitions a copy of those of \( s_1 \),
  and incoming transitions the union of those of \( s_1 \) and \( s_2 \).
}

R-equivalent(States \( s_1, s_2 \))
{
  if \( \text{finalState}(s_1) \wedge \text{not finalState}(s_2) \) or \( \text{not finalState}(s_1) \wedge \text{finalState}(s_2) \)
    return false
  else return true iff
  \( \forall t \in T \cdot \text{source}(t) = s_1 \Rightarrow \exists t' : T \cdot \text{source}(t') = s_2 \wedge \text{trigger}(t') = \text{trigger}(t) \wedge \text{action}(t') = \text{action}(t) \)
  \( \wedge \text{guard}(t') \equiv \text{guard}(t) \wedge \text{target}(t') \sim \text{target}(t) \)
  \( \forall t \in T \cdot \text{source}(t) = s_2 \Rightarrow \exists t' : T \cdot \text{source}(t') = s_1 \wedge \text{trigger}(t') = \text{trigger}(t) \wedge \text{action}(t') = \text{action}(t) \)
  \( \wedge \text{guard}(t') \equiv \text{guard}(t) \wedge \text{target}(t') \sim \text{target}(t) \)

  // Where \( s_1 \sim s_2 \) means the targets are the same state, or one is \( s_1 \) and the other \( s_2 \).
}

G-mergeEquivalentStates(EFSM \( M = (S, T, A, S_i) \))
{
  For a set of transitions \( T, \text{triggers}[T] \) denotes the set of triggers of the transitions of \( T \).
  Function \( \text{metric}(g) \) denotes the number of states removed by merging groups in \( g \) into single states: \( \Sigma_{s \in g}|s| - 1 \)
  where \(|s|\) denotes the size of set \( s \).
  \( \text{possibleGroups} = \{ ss \in 2^S | |ss| \geq 2 \wedge \)
  \( (\forall t \in T \cdot \text{source}(t) \in ss \wedge \text{target}(t) \in ss \Rightarrow \text{action}(t) = \text{skip} ) \)
  \( \wedge ( (\text{triggers}\{t \in T | \text{source}(t) \in ss \wedge \text{target}(t) \in ss\} ) \)
  \( \cap \text{triggers}\{t \in T | \text{source}(t) \in ss \wedge \text{target}(t) \notin ss\} = \emptyset \} ) \)
  if \( \text{possibleGroups} \neq \emptyset \) then
    \( \text{mergeGroups} = \{ pg \in \text{possibleGroups} | pg \neq \emptyset \wedge \forall \text{state sets } ss_1, ss_2 \in pg \cdot ss_1 \neq ss_2 \Rightarrow ss_1 \cap ss_2 = \emptyset \} \)
    \( \text{bestGroup} = g \in \text{mergeGroups} \text{ such that } \text{metric}(g) \text{ is maximal}. \)
    \( \text{collapseStates}(M, \text{bestGroup}) \)
}

Figure 5: Algorithms \( A_3 \) and \( A_4 \): Two State Merging algorithms
4. ALGORITHM CORRECTNESS

In what follows we show that the basic slicing Algorithm $A_1$ is correct with respect to the strong semantic requirement. Moreover, we show that the constant propagation and right-equivalent merging algorithms, $A_2$ and $A_3$, are also correct with respect to this requirement. Finally, the group merging algorithm is shown to be correct with respect to the weak semantic requirement. Before we present the proofs of correctness, we describe the semantics of EFSMs, formally.

Definition 5 (Configurations)
Let $\Sigma$ be the set of all possible stores for an EFSM. For EFSM $M = (S, T, A, S_0)$, a configuration is a pair $(s, \sigma)$ where $s \in S$ is a state and $\sigma \in \Sigma$ is a store.

Definition 6 (Status)
For EFSM $M = (S, T, A, S_0)$, a status is a pair $(C, E)$ where $C$ is a configuration and $E \in A^*$ is an event sequence. A status is an initial status if the state contained in its configuration is an initial state.

Definition 7 (Enabled Transitions)
For Status $St = (C, E)$ of an EFSM $M = (S, T, A, S_0)$ where $C = (s, \sigma)$, transition $t$ is enabled iff $source(t) = s$, guard$(t)$ evaluates to True when evaluated using $\sigma$, and trigger$(t) = head(E)$. The set $Enabled(St)$ denotes the set of all transitions enabled in Status $St$.

The semantics of an EFSM $M$, is formalized as a Labeled Transition System (LTS). States of the LTS are the statuses of $M$ and transitions of the LTS have the same labels as the transitions of $M$ but extended with an additional label for implicit transitions (see the operational semantics definition below). The LTS has a labeled step rule which defines legal transitions. A closed system assumption is made in which the status is only changed explicitly by the step rule. The semantics of EFSM $M$ taking enabled transition $t$ is given as an LTS move from Status $St_1$ to Status $St_2$, denoted $M \vdash St_1 \xrightarrow{t}{\text{LTS}} St_2$. In doing so the store is updated using the store update function denoted by $[\text{action}(t)]$. This single step rule is formalized as follows:

Definition 8 (LTS Step $\xrightarrow{\text{LTS}}$)
For Status $St = ((s, \sigma), E)$ of EFSM $M$, taking enabled transition $t$ produces the next status:

$$t \in Enabled(St) \land \sigma' = [\text{action}(t)]\sigma \quad \text{where} \quad M \vdash ((s, \sigma), E) \xrightarrow{t}{\text{LTS}} ((\text{target}(t), \sigma'), \text{tail}(E))$$

Definition 9 (Operational Semantics)
The operational semantics defines the meaning of EFSM $M = (S, T, A, S_0)$ as the LTS $(ST, ST_i, L, \xrightarrow{\text{LTS}})$ where $ST$ and $ST_i$ are sets of statuses and initial statuses, respectively, the label set $L$ is $T \cup \{\varepsilon\}$, where the special label $\varepsilon$ is used to model implicit transitions, and $\xrightarrow{\text{LTS}}$ is the LTS step rule from Definition 8.

Definition 10 (Stuttering Rule)
For a Status $St = (C, E)$, if no transitions are enabled, then the head of the event sequence is consumed without changing $C$.

$$\text{Enabled}(St) = \emptyset \quad \text{where} \quad M \vdash (C, E) \xrightarrow{\varepsilon}{\text{LTS}} (C, \text{tail}(E))$$
Definition 11 (Reachable Statuses Set)
For EFSM $M = (S, T, A, S_1)$ having LTS $(St, St_{1}, L, \rightarrow_{LTS})$ and a restricted event alphabet $A_R$, the set $\text{Statuses}(M, A_R)$ is the set of statuses reachable in zero or more steps from an initial status in an environment that produces only events found in $A_R$:

$$\text{Statuses}(M, A_R) \equiv \{ st \in St \mid st_{1} \in St_{1} \land M \vdash st_{i} \xrightarrow{E} st \text{ where trigger}(l) \in A_R \}$$

Definition 12 (Status Congruence $\cong$)
Statuses $((s_1, \sigma_1), E_1)$ and $((s_2, \sigma_2), E_2)$ are congruent, written $((s_1, \sigma_1), E_1) \cong ((s_2, \sigma_2), E_2)$ iff $\sigma_1 = \sigma_2$ and $E_1 = E_2$.

In the following proofs, $M'$ continues to denote a slice of EFSM $M$. Subscripts are added to denote a slice produced by a particular slicing algorithm; thus, a slice of $M$ produced by $A_1$ (Figure 3) is denoted $M_{1}'$. The main theorem uses simulation to establish the correctness of Algorithm $A_1$ by showing that for all event sequences $E \in f(A^*, T)$ $M$ and its slice $M_1'$ produce corresponding statuses. That is, whatever $M$ can do using only the restricted event set $M'$ also can do. This result is then extended to show that the simulation still holds for $A_2$ slices. Finally, it is shown that $A_2$ produces slices who’s simulation is homomorphic to those for slices produced by $A_2$ and, for event sequences in $f(\text{ift}(M, A^*), T)$. This result extends to slices produced by $A_1$.

Simulation is a relation between statuses in the operational semantics of a pair of EFSMs. We write $St_1 \triangleright Sl_{t_2}$ to mean $St_2$ simulates $St_1$. We mildly abuse the word simulation when saying $M'$ simulates $M$ for a given set of event sequences $E$. The intuition is that if $M'$ simulates $M$ for $E$ then the two machines have the same behavior when placed in an environment that can only produce event sequences in $E$. First we give the general definition of a simulation between the semantics of EFSMs and then the definition for the meaning of $M'$ simulates $M$ for $E$.

Definition 13 (Simulation relation $\triangleright$ for the operational semantics of EFSMs)
Let EFSMs $M = (S, T, A, S_1)$ and its slice $M' = (S', T', A', S'_1)$ have statuses $(C_1, E_1)$ and $(C'_1, E'_1)$ respectively. Then $(C_1, E_1) \triangleright (C'_1, E'_1)$ if whenever

$$(C_1, E_1) \cong (C'_1, E'_1) \land M \vdash (C_1, E_1) \xrightarrow{E} (C_2, E_2)$$

there exists a status $(C'_2, E'_2)$ such that

$$M' \vdash (C'_1, E'_1), E_2 \rightarrow_{LTS} (C'_2, E'_2)$$

and $(C_2, E_2) \triangleright (C'_2, E'_2)$ where $E_2$ denotes $\text{tail}(E_1)$.

Definition 14 (EFSM simulation)
EFSM $M' = (S', T', A', S'_1)$ simulates EFSM $M = (S, T, A, S_1)$ for $E$, written $M \triangleright_{E} M'$, if

- there is a simulation relation $\triangleright$, between the two semantics whose range includes all statuses in the semantics of $M'$ for event sequences in $E$ and
- for every start state $s'_1 \in S'_1$ for $M'$ there is a corresponding start state $s_1 \in S$, for $M$ so that any event sequence $E \in E$ and store $\sigma$, $((s_1, \sigma), E) \triangleright (s'_1, \sigma, E)$

The first theorem states that projecting the behavior of an EFSM via an ignore set is safe as the resulting EFSM simulates the original with respect to the reduced environment.

Theorem 1 Given EFSM $M = (S, T, A, S_1)$ and its slice $M'_1 = (S'_1, T'_1, A', S'_1)$ produced by algorithm $A_1$ using ignore set $T$ $M \triangleright_{E} M'_1$ for $E = (A - T)^*$. ($M'_1$ simulates $M$ for $E = (A - T)^*$).
and the resulting configurations are identical, satisfying the simulation relation. Again $\triangleright_{2}$ is defined similarly to $\triangleright$ and the argument for the initial configurations of $M'_{2}$ satisfying the relation is similar. □

We now show that the simulation produced by Algorithm $A_{3}$ is homomorphic to that produced by Algorithm $A_{2}$.

**Proposition 2** Given EFSM $M = (S, T, A, S_{i})$ and its slice $M'_{1} = (S_{1}', T_{1}', A_{1}', S_{1}')$ produced by $A_{3}$ using ignore set $I$, $M \triangleright_{E} M'_{1}$ where $E = \{A - I\}$.

**Proof.** Let $M'_{2}$ be the slice of $M$ produced by $A_{2}$ using $I$. We show that there exists a (model) homomorphism, $h$, from $M_{2}$ to $M'_{2}$. We then argue that $h$ naturally extends to a homomorphism between the simulations of $M'_{2}$ and $M'_{2}$; thus, $M \triangleright_{E} M'_{2}$ follows from $M \triangleright_{E} M'_{1}$.

Assume that $M'_{2}$ includes states $s_{1}, s_{2}$ where $s_{1}$ and $s_{2}$ have identical outgoing transitions for events in $A - I$. In more detail, for each transition $t_{1}$ where $\text{source}(t_{1}) = s_{1}$ and $\text{lbl}(t_{1}) = e_{1}$ and $\text{target}(t_{1}) = x$, there is a transition $t_{2}$ where $\text{source}(t_{2}) = s_{2}$, $\text{lbl}(t_{2}) = e_{2}$, and $\text{target}(t_{2}) = x'$ where $x = x'$ or $\{x, x'\} = \{s_{1}, s_{2}\}$. Model $M_{1}$ is identical to $M_{2}$ except that $s_{1}$ and $s_{2}$ are replaced by a single state $s$, each transition $t_{1}$ targeting $s_{1}$ or $s_{2}$ is replaced by a transition targeting $s$. Matched transitions $t_{1}$ and $t_{2}$ in $M'_{2}$ as above are replaced in $M'_{1}$ by a single transition $t$ with $\text{source}(t) = s$, $\text{lbl}(t) = e_{1}$ and $\text{target}(t) = z$ where $z = z'$ if $x \neq s_{1}$ and $x \neq s_{2}$, or $z = s$ otherwise. Therefore, $M'_{1}$ is clearly homomorphic to $M'_{2}$.

This model homomorphism induces a simulation homomorphism that maps states for $M_{2}$ to the slice to the states for $M'_{2}$. In particular, the states for $s_{1}$ and $s_{2}$ are mapped those for $s$. Since the associated transitions $t_{1}$ and $t_{2}$ have the same label, the resulting configurations are not affected. Consequently, $M \triangleright_{E} M'_{1}$ implies $M \triangleright_{E} M'_{2}$. □

Finally we show that applying $A_{3}$ also induces a simulation between the semantics of the original machine and the resulting slice, but only with some further restriction on the type of input event sequences that can be considered in the semantics.

**Proposition 3** Given EFSM $M = (S, T, A, S_{i})$ and its slice $M'_{1} = (S_{1}', T_{1}', A_{1}', S_{1}')$ produced by $A_{3}$ using ignore set $I$, $M \triangleright^{G}_{E} M'_{1}$ where $E = \{\text{if}(M, A') - I\}$ and the set of restricted sequences which are stutter free on $M$.

**Proof sketch.** Let $M'_{1}$ be the slice of EFSM $M$ produced by applying algorithm $A_{3}$. Let $h$ denote the model homomorphism the exists between $M_{1}'$ to $M_{1}''$. Proposition 2 shows that there is a simulation $\triangleright^{G}_{E}$ such that $M \triangleright^{G}_{E} M'_{1}$. (The proposition actually proves the stronger result for $E = \{\text{if}(M, A') - I\} = (A - I)^{∗}$ which is a superset of $E$.) The remainder of the proof shows that using $h$ this relationship extends to $M'_{2}$ (i.e., that $M \triangleright^{G}_{E} M'_{2}$).

Model homomorphism $h$ induces a homomorphism between the LTSs that are the semantics of $M'_{1}$ and $M'_{2}$ respectively but only if the input event sequences for $M'_{1}$ are stutter free. Consider a status $((s, \sigma), E)$ in the semantics of $M'_{1}$ and its image in the semantics of $M'_{2}$ under the induced homomorphism (also called $h$) is $((h(s), \sigma), E)$. Now suppose $M'_{1} \triangleright((s, \sigma), E) \xrightarrow{LT S} ((s', \sigma'), E')$ where $E' = \text{tail}(E)$. Since $E$ is stutter free for $M'_{1}$ the label on $t$ has trigger $(t) = \text{head}(E)$. We have two cases to consider. In the first case, head$(E)$ is internal and $M'_{1}$ takes a self-loop: $M'_{1} \triangleright((h(s), \sigma), E) \xrightarrow{LT S} (h(s'), \sigma'), E'$ and $h(s) = h(s')$ and $\sigma' = \sigma$ (because internal labels have no actions). Alternatively in the second case head$(E)$ is exiting and $M'_{1} \triangleright((h(s), \sigma), E) \xrightarrow{LT S} (h(s'), \sigma'), E'$ because G-mergeEquivalentStates preserves exiting transitions. Note that if $E$ is not stutter free on $M'_{1}$ this breaks down. It is possible that $M'_{2}$ stutters: $M'_{2} \vdash ((s, \sigma), E) \xrightarrow{LT S} ((s, \sigma), E')$ whereas because of the merging of states there exist an exiting transition from $(h(s))$ in $M'_{2}$ that is triggered by head$(E)$.

Thus, in conclusion, homomorphism $h$ induces a homomorphism on simulations such that for event sequences in $\text{if}(M, A') - I$, $M'_{2} \triangleright^{h(\cdot)}((\cdot), \cdot)$. Combined with the result from Proposition 2 that $M \triangleright^{G} M'_{2}$ we conclude that $M \triangleright^{h(\cdot)}((\cdot), \cdot)$.
Figure 7: All slice sizes for Algorithms $A_1$, $A_2$, $A_3$ and $A_4$. The upper two figures show all slice sizes when the ignore set $I$ is set to each of the 142 events and the lower two graphs show all slice sizes when the ignore set $I$ is set to each of the 1323 pairs of events.

Figure 8: All slice sizes for Algorithms $A_1$, $A_2$, $A_3$ and $A_4$ using all possible ignore sets containing all but one ($n - 1$) events.

Figure 7 shows graphs of all slice sizes for all ignore sets of size $n = 1$ and $n = 2$ using all four algorithms. In each graph, the $x$-axis shows the slices in monotonically increasing order of slice size. Normalized to a percentage, the $y$-axis shows slice size as a percentage of the number of states or the number of transitions. The data is individually sorted, and thus should not be used to compare a particular slice. In all graphs, the four lines show similar trends. Considering the extremes, for Algorithm $A_1$ and singleton event ignore-sets, there are no states removed in approx. 80 of the 142 slices (indicated by the straight line on the top right of the diagram). For these slices, the reductions in the number of transitions is small (less than 10%). At the other end of the spectrum, in seven cases, the slices have zero transitions. This occurs when a key event is used as the slicing criterion. Further investigation reveals that these events often occur on the transitions close to an initial state where the triggering event determines if further processing by the model is needed. For example, ignoring the Card() event in the ATM model corresponds to not inserting an ATM card, and thus the ATM remains in its idle state.

### 5.2 Large Ignore Sets

The second research question considers larger ignore sets. For $A_1$, using ignore sets of size $n = 8$, a larger size than considered in Table 2, the average slice includes 27.6% of the states and 10.4% of the transitions. The sizes for $A_2$, $A_3$, and $A_4$ follow the same pattern as seen in Table 2. At the extreme, ignoring all but one event produces an average slice size of 12.7% states and 1.1% transitions. Figure 8 show graphs of these slices for all ten models where only a single line is visible for all four algorithms because they all produce very similar sized slices.

One motivation for this research question was to support a comparison with dependence-based EFSM slicing techniques and with the slicing of programs. Dependence-based techniques extract the part of an EFSM that affects one or more elements (states, transitions, or variables used in transition labels) from the EFSM. In the following comparison these elements are assumed to be transitions. Previous studies of such slices using the same ten subjects report an average slice size of between 38.4% and 68.0% of the transitions using a variety of definitions of “dependence” [2]. From a program $P$, a common variant of program slicing extracts a semantically meaningful 'subprogram' that captures the execution of a particular statement from $P$. 

#### Table 2: Average slice size when ignoring $n$ events.

<table>
<thead>
<tr>
<th>States</th>
<th>Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n=1$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>85.3%</td>
</tr>
<tr>
<td>$A_2$</td>
<td>80.5%</td>
</tr>
<tr>
<td>$A_3$</td>
<td>78.6%</td>
</tr>
<tr>
<td>$A_4$</td>
<td>71.2%</td>
</tr>
</tbody>
</table>
It is difficult to make a direct comparison of environment-based slicing with the slicing of programs or dependence-based techniques for EFSMs because the slicing criteria are so different: environment-based slicing starts with one or more events to ignore, while the others start with a particular transition or statement to preserve. The average program slice size is about one third of a program [4]. This is comparable to the reduction in transitions for a model using an ignore set of size 4.

Comparing with EFSM dependence-based techniques, one possible comparison clearly favors environment-based slicing: given that dependence-based approaches preserve the behavior of a single transition, one might compare them with an ignore set that includes all but one event. Considering all but one event is similar to dependence-based slicing’s seeking to preserve a single transition from the model. In this case, $A_4$’s average slice size of 1.0% of the transitions compares quite favorably. A similar result might be expected from comparing the use of ignore sets of size one with dependence-based slices taken with respect to all transitions not using an ignored event. As this slicing criterion (starting point) is a substantial portion of the model, this comparison too favors environment-based slicing. Finally, consider a like-for-like comparison using a single transition for dependence-based slicing and an ignore set of size one for environment-based slice. Here, $A_4$’s average slice size of 66.1% of the transitions, while toward the large end of the range for dependence-based slicing (28.4% – 68%), still provides a comparable reduction.

### 5.3 Runtime Efficiency

The final research question considers the runtime efficiency of the slicer. Slices were computed on a 2.4GHz Intel(R)Core Duo CPU having 4GB memory. For ignore sets of size one, the results show that $A_1$ and $A_2$ are extremely fast with maximal times of 0.016s and 0.042s, respectively. In contrast, the execution time for $A_3$ and $A_4$ reflect the PSPACE complexity of minimizing an EFSM (and in general an NFA) [18, 13]). The average times were 27s and 36s, and the maximal times were 94s and 753s, respectively. All times drop with larger ignore sets. For example, the worst-case times for the four algorithms using all ignore sets of size 4 are 0.0012s, 0.025s, 29s, and 126s, respectively. While all four show a large percentage drop, the impact is more meaningful for $A_3$ and $A_4$, where the worst case run time for $A_3$ and $A_4$ drops by 70% and 83%, respectively.

### 6. RELATED WORK

Our model projections are related to model dependence analysis and slicing. However, previous work on slicing state-based formalisms [17, 21, 23, 24, 33] has concentrated exclusively on a white-box view of a state machine, in which the machine is much like a special case of a program; its states loosely play the role of states and its transitions merely mimic control flow. This has allowed concepts from program slicing to be adapted for state-based models. Though valuable, such approaches miss the ability to talk about sub-models extracted from an original model in terms of the environment in which the model operates. In this section, we describe the relationship between model projection and previous work on dependence analysis and slicing of state-based models.

Korel et al. [21] describe the increased complexity of the proof obligation for model-based slicing, arising from the presence of environmental considerations, while Sanchez et al. [30] advocate a clear separation between equipment control activities and product manufacturing procedures. However, neither provide algorithms that yield model projections defined for restricted environments.

Cheung and Kramer [7] consider a form of reachability analysis in terms of events. Such an analysis is a pre-requisite of slicing, which we use in our model projection. However, Cheung and Kramer use their approach to simplify interfaces, not to simplify the models themselves. The two complement each other as model projection computes the implications of the simplified interfaces on the models themselves.

The previous work that most closely resembles model projection is that described by Eshuis and Wieringa [11]. Their approach was developed to handle the state-space explosion problem when model checking workflows are modelled as UML activity diagrams. Rather than being based on dependence analysis, their approach consists of four reduction rules based on activity hypergraphs. Each of their four rules is related to both model-based slicing and aspects of model projection. The first of the four rules reflects changes made to the machine during the overall rewrite process back to its environment. This is similar to our notion of implicit-transition-free event sequences (Definition 2). The second rule insists that no two named events can occur at the same time (i.e., they can only be interleaved). This is similar to our requirement that only one event is dealt with at a time. The third rule defines when a variable can be removed, and is similar to a data dependence analysis that is a prerequisite of dependence-based slicing. The fourth rule, which resembles a restricted form of slicing, defines how wait and activity states can be removed from UML activity hypergraphs. In comparison, with our approach, the environmental restriction is specified by the designer, whereas in the approach of Eshuis and Wieringa, the iterative application of the four rules can have the effect of removing some environmental elements.

Model projection introduces the possibility of slicing on the events in which a model participates to produce a reduced model specialized to a specific environment. Model projection therefore presents new challenges and opportunities for model-based slicing, in which the environment essentially becomes a first class citizen in the slicing criteria.

### 7. CONCLUSION

We introduced model projection in order to extend to the design level benefits that slicing offers at the coding level. However, modelling languages are not simply higher-level programming languages and so we found that we needed to re-think the way in which the simplification inherent in slicing is defined and implemented. In particular, we focused on the model’s environment as a criterion for simplification, rather than the internal white box structure of the model itself.

As with any new approach there is an obligation to demonstrate correctness, effectiveness, and efficiency and these have therefore formed the primary contributions of this paper. We have introduced and proved correct a set of four algorithms for model projection that trade effectiveness (in terms of projected model size) for efficiency (in terms of projection construction time). The implementations of our four algorithms are available from the SLIM project website at http://slim.dcs.kcl.ac.uk.

### 8. REFERENCES


