Problem 1: Determine whether the following argument form is valid.

\[
\begin{align*}
& p \rightarrow q \\
& q \rightarrow \neg p \\
& p \lor r \\
\therefore r
\end{align*}
\]

Problem 2: Deduce the conclusion below from the given premises.

\[
\begin{align*}
& \neg p \\
& s \lor t \rightarrow p \\
& q \rightarrow s \\
& q \lor \neg t \rightarrow r \\
& \neg q \land r \rightarrow u \\
\therefore u
\end{align*}
\]

Problem 3: Design a circuit with three inputs whose output is on if and only if none of the inputs are on or exactly two of the inputs are on.

Problem 4: Design a circuit that, given four 1-bit numbers as input, computes their 3-bit sum.

Problem 5: Write negations of each of the following statements.

(a) Either ER or Scrubs is on NBC.

(b) Futurama is on TV as long as the football game did not run late.

(c) M*A*S*H is on Tuesday and either Benson or Blossom is on Monday.

(d) All shows on Thursday are popular.

(e) Some shows on NBC are about lawyers.
Problem 6: Let $F(x, y)$ be the predicate “$x$ has been to $y$” and $E(x, y)$ be the predicate “$x$ likes to eat $y$”. Assume that the predicates are true for the values indicated by the tables below.

<table>
<thead>
<tr>
<th></th>
<th>Boston</th>
<th>New York</th>
<th>DC</th>
<th>Frederick</th>
<th>shrimp</th>
<th>beef</th>
<th>kohlrabi</th>
<th>tofu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tom</td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td>Tom</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bob</td>
<td></td>
<td>x</td>
<td></td>
<td>Bob</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leo</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Translate each of the following statements to predicate logic and determine whether they are true or false.

(a) Pat has been to Frederick.
(b) Everyone likes to eat beef.
(c) No one likes to eat tofu.
(d) Someone who has been to Boston likes to eat shrimp.
(e) Everyone who likes to eat kohlrabi has been to DC.
(f) At least two people have been to New York.
(g) Every city has been visited by some person who likes to eat shrimp.
(h) No one has been to every city.
(i) There is a food liked by everyone who has been to a certain city.

Problem 7: Prove or disprove:

(a) $n^2 + 5n + 1$ is composite for all positive integers $n$.
(b) The product of any three odd numbers is odd.
(c) For all integers $x$ and $y$, if $x | y$ and $y | x$ then $x = y$. 