<table>
<thead>
<tr>
<th>( p \land r )</th>
<th>( p \lor q )</th>
<th>( p \lor r )</th>
<th>( p \lor (q \land r) )</th>
<th>( (p \lor q) \land r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
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<td>F</td>
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<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

\[
p \rightarrow (q \rightarrow r) \equiv (p \rightarrow q) \rightarrow r
\]
\( p \) unless \( \sim q \) \( \quad \sim q \rightarrow p \)

for \( p \), it is necessary that \( q \) \( \quad p \rightarrow q \)

for \( p \), it is sufficient that \( q \) \( \quad q \rightarrow p \)

for \( p \), it is necessary and sufficient that \( q \) \( \quad p \iff q \) “biconditional”

\[ = (p \rightarrow q) \land (q \rightarrow p) \]

\[
\begin{array}{c|c|c}
  p & q & p \iff q \\
  \hline
  T & T & T \\
  T & F & F \\
  F & T & F \\
  F & F & T \\
\end{array}
\]
\[(\neg (p \lor q) \lor \neg (p \land q)) \equiv \neg p\]

by DeMorgan's Law

\[\neg p \lor (\neg q \land \neg p)\]

by Double Negation

\[\neg p \lor (q \lor \neg q)\]

by Distributive Law (backwards)

\[\neg p \lor \neg p\]

by Negation

\[\neg p\]

by Identity
\[
\begin{align*}
&\{ g \rightarrow g \} \text{ premises} \\
&\therefore p \quad \text{ conclusion}
\end{align*}
\]

Argument is valid if conclusion is true whenever the premises are true.

Above argument form is invalid b/c it's possible for conclusion to be F when premises both T:

<table>
<thead>
<tr>
<th>P</th>
<th>G</th>
<th>P \rightarrow G</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Conclusion F when premises T invalid

Critical rows:

P \rightarrow G

Premises all T

(T F T)

(F F F)

(T T T)
\[
\begin{align*}
\text{premises} & \quad \text{conclusion T on all critical rows} \\
T \quad q & \quad \therefore q \\
T \quad F & \quad F \\
F \quad T & \quad T \\
F \quad F & \quad T
\end{align*}
\]

"modus ponens"

\[
\begin{align*}
\text{p} & \quad \text{p} \lor \text{q} \quad \text{p} \lor \text{q} \quad \text{p} \lor \text{q} \quad \text{p} \lor \text{q} \\
\therefore \text{p} & \quad \therefore \text{p} \quad \therefore \text{p} \quad \therefore \text{p} \\
\therefore \text{p} & \quad \therefore \text{p} \quad \therefore \text{p} \quad \therefore \text{p}
\end{align*}
\]

\[
\begin{align*}
p \rightarrow q \equiv \neg q \rightarrow \neg p & \quad \therefore \text{p} \rightarrow q \equiv \neg q \rightarrow \neg p \\
\therefore \text{p} \rightarrow q \equiv \neg q \rightarrow \neg p & \quad \therefore \text{p} \rightarrow q \equiv \neg q \rightarrow \neg p
\end{align*}
\]

Contrapositive

"converse"

"inverse"
1) \( p \rightarrow \neg q \)  
2) \( r \rightarrow q \)  
3) \( p \)  
4) \( \neg r \)  
5) \( t \rightarrow u \)

6) \( \neg q \)  (modus ponens on 1, 3)  
7) \( \neg r \)  (modus tollens on 2, 6)  
8) \( s \)  (elimination, 4 and 7)