16d) No logicians are lazy

\[ \forall x \in L, -\exists(x) \exists(x) = "x \text{ is lazy}" \]

\[ L = \text{set of logicians} \]

f) -1 is not the square of any real number

\[ \forall x \in \mathbb{R}, -1 \neq x^2 \]

26b) The product of any two odd integers is odd

\[ \forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, (x \text{ is odd } \land y \text{ is odd}) \implies xy \text{ is odd} \]

\[ \forall x \in \mathbb{Z} \]
27b) \( \exists x \text{ s.t. } (\text{Rect}(x) \land \neg \text{Square}(x)) \)  

There exist rectangles but no squares. 😕

\( \exists x \text{ s.t. } \text{Rect}(x) \land \neg \exists x \text{ s.t. } \text{Square}(x) \) 

There is a rectangle that is not a square ✓

Exam #1 Fri Oct 7th

Chs 1, 2 ✓ mostly
\( \S 3.1 - 3.3 \)

\( \forall x, P(x) \rightarrow Q(x) \)

\( \forall x, P(x) \)
**Thm:** The sum of two even integers is even.

\[ \forall x, y \in \mathbb{Z}, \ (x \text{ is even } \land y \text{ is even}) \Rightarrow (x + y) \text{ is even} \]

To prove: start w/ generic \( x, y \in \mathbb{Z} \) show that

\[ (x \text{ is even } \land y \text{ is even}) \Rightarrow x + y \text{ is even} \]

**Def:** For integers \( x \), "\( x \) is even" means

\[
\frac{x}{2} \text{ has no remainder}
\]

\[
\frac{x}{2} \in \mathbb{Z}
\]

\[
\exists k \in \mathbb{Z} \text{ s.t. } k = \frac{x}{2}
\]

\[
\exists k \in \mathbb{Z} \text{ s.t. } x = 2k
\]
Thm: \( \forall x, y \in \mathbb{Z}, \ (x \text{ is even } \land y \text{ is even}) \rightarrow x + y \text{ is even} \)

Proof:

Assume \( x, y \in \mathbb{Z} \) [want to prove \( (x \text{ is even } \land y \text{ is even}) \rightarrow (x + y \text{ even}) \)]

Assume \( x \text{ is even } \land y \text{ is even} \)

\[ \exists k \in \mathbb{Z} \text{ s.t. } x = 2k \quad \text{(def of even)} \]

\[ \exists l \in \mathbb{Z} \text{ s.t. } y = 2l \quad \text{(def of even)} \]

Let \( k, l \) be as above \( \text{(Axiom of choice)} \)

Then \( x + y = 2k + 2l = 2(k + l) \quad \text{(substitution)} \)

where \( k + l \in \mathbb{Z} \quad \text{(algebra)} \)

\[ \exists m \in \mathbb{Z} \text{ s.t. } x + y = 2m \quad \text{(by closure of \( \mathbb{Z} \) under +)} \]

(namely \( m = k + l \))

\[ \therefore x + y \text{ is even} \]
\( \forall x, y \in \mathbb{Z}, (x \text{ even } \land y \text{ even }) \rightarrow x + y \text{ is even} \)

**Thm:** The product of two even integers is even.

\( \forall x, y \in \mathbb{Z}, x \text{ even } \land y \text{ even } \rightarrow xy \text{ even} \)

**Proof:** Assume \( x, y \in \mathbb{Z} \) and both are even.

[want \( xy \) is even]

Then \( \exists k \in \mathbb{Z} \text{ s.t. } x = 2k \) and \( \exists l \in \mathbb{Z} \text{ s.t. } y = 2l \) \( \text{(def of even)} \)

Let \( k, l \) be as above \( \text{(Ar of choice)} \)

Then \( xy = 2k \cdot 2l = 2 \cdot (2kl) \) \( \text{(sub, c.l.g., closure of } \mathbb{Z} \text{ under \times)} \)

\( \exists m \in \mathbb{Z} \text{ s.t. } xy = 2m \) \( \text{(namely } m = 2kl ) \)

\( \therefore xy \text{ is even } \) \( \text{(def of even)} \)
\[ \forall x, y \in \mathbb{Z}, (x \text{ even } \land y \text{ even }) \rightarrow xy \text{ is even} \]

Def: \( X \) is odd iff \( \exists k \in \mathbb{Z} \text{ s.t. } x = 2k + 1 \)

\( x \) is prime iff only divisors are 1 \& itself

if something divides \( x \), then something is 1 \& \( x \)

\( x \) is composite iff \( x \) has non-trivial factors

\( \exists y, z \in \mathbb{Z}^+ \text{ s.t. } yz = x \land y \neq 1 \land z \neq 1 \)

\( x \) is prime \( \sim (\exists y, z \text{ s.t. } yz = x \land y \neq 1 \land z \neq 1) \)

\( \forall y, z, \sim (\gamma z = x \land y \neq 1 \land z \neq 1) \)
For all integers $n \geq 1$, $n^2 + 3n + 2$ is not prime.

Proof: Let $n \in \mathbb{Z}^+$. [Need $n^2 + 3n + 2$ is not prime]