§4.4 #8a)

$h_0 = 1 \quad h_1 = 2 \quad h_2 = 3$

$h_k = h_{k-1} + h_{k-2} + h_{k-3}$

	3^n
$h_3 = 6$	27
$h_4 = 11$	81
$h_5 = 20$	243

Thm: $\forall n \in \mathbb{Z}, n \geq 0 \rightarrow h_n \leq 3^n$

Proof: Base cases $(n=0)$ $\quad h_0 = 1 \quad 3^0 = 1 \quad 1 \leq 1$ so $h_0 \leq 3^0$

$(n=1) \quad h_1 = 2 \quad 3^1 = 3 \quad 2 \leq 3$ so $h_1 \leq 3^1$

$(n=2) \quad h_2 = 3 \quad 3^2 = 9 \quad 3 \leq 9$ so $h_2 \leq 3^2$

Ind step: Assume $k > 2$ and $\forall i \in \mathbb{Z}, 0 \leq i < k \rightarrow h_i \leq 3^i$

[want $h_k \leq 3^k$]

Then $h_k = h_{k-1} + h_{k-2} + h_{k-3} \leq 3^{k-1} + 3^{k-2} + 3^{k-3}$

since by ind hyp, $h_{k-1} \leq 3^{k-1}$, $h_{k-2} \leq 3^{k-2}$, and $h_{k-3} \leq 3^{k-3}$

$a_1 \leq b_1$

$a_2 \leq b_2$

$a_3 \leq b_3$

$a_1 + a_2 + a_3 \leq b_1 + b_2 + b_3$

Also, $3^{k-1} + 3^{k-2} + 3^{k-3} \leq 3^{k-1} + 3^{k-1} + 3^{k-1}$
$= 3 \cdot 3^{k-1} = 3^k$

$\therefore h_k \leq 3k$.

24)

$P(0), P(1), P(2)$ all T

$\forall k \in \mathbb{Z}, (k \geq 0 \land P(k)) \rightarrow P(3k) \qquad \underset{=}{1 \geq 0} \land P(1) \rightarrow P(3)$

$P(0)$

$P(0) \rightarrow P(0)$ by univ inst $(k=0)$

$P(1) \rightarrow P(3)$

$P(1)$

$\therefore P(3)$

$P(3) \rightarrow P(9)$

$\therefore P(9)$

$P(2) \rightarrow P(6)$

$P(2)$

$\therefore P(6)$

2) $b_1 = 4 \quad b_2 = 12 \quad k \geq 3, \; b_k = \underline{b_{k-1}} + \underline{b_{k-2}}$

Thm: $\forall k \in \mathbb{Z}, \; k \geq 1 \longrightarrow 4 \mid b_k$

Proof: Base cases $(n=1)$ $b_1 = 4, \; 4 \mid 4$, so $4 \mid b_1$

$(n=2)$ $b_2 = 12, \; 4 \mid 12$, so $4 \mid b_2$

Ind step: Assume $k > 2$ and $\forall i \in \mathbb{Z}, \; 1 \leq i < k \rightarrow 4 \mid b_i$

[want $4 \mid b_k$]

by ind hyp, which applies since $1 \leq k-1, \; k-2 < k$

$a \mid b \wedge a \mid c \rightarrow a \mid b+c$

$b_k = b_{k-1} + b_{k-2}$ where $4 \mid b_{k-1}$ and $4 \mid b_{k-2}$

$\therefore$ by thm, $4 \mid b_{k-1} + b_{k-2}$, so $4 \mid b_k$.

$A = \{ x \in D \mid P(x) \}$ means $x \in A \longleftrightarrow x \in D \wedge P(x)$

Ex: $A = \{ x \in \mathbb{Z}^+ \mid x < 4 \} = \{ 1, 2, 3 \}$

$B = \{ x \in \mathbb{Z} \mid \underline{x^2 < 10} \} = \{ -3, -2, -1, 0, 1, 2, 3 \}$

$A \subseteq B$ means $\forall x, \; x \in A \rightarrow x \in B$

"A is a subset of B"

(proper subset is a subset but not equal)

$A = B$ means $\forall x, \; x \in A \longleftrightarrow x \in B$

$\equiv$

$A \subseteq B \wedge B \subseteq A$

$2 \in \{1, \underline{2}, 3\}$ T

$\underline{2} \subseteq \{1, 2, 3\}$ not a set F

$\{\underline{2}\} \subseteq \{1, \underline{2}, 3\}$

$\{2\} \in \{1, 2, 3\}$ F

$\{2\} \in \{1, 2, 3, \underline{\{2\}}\}$ T

$A \cup B = \{x \mid x \in A \vee x \in B\}$

"union of A, B"

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

"intersection of A, B"

$B - A = \{x \mid x \in B \wedge x \notin A\}$

"set difference"

$A^c = \{x \in \underline{U} \mid x \notin A\}$

universe

$\emptyset = \{\} = \{x \mid c\}$ $\quad x \in \emptyset$ is always F

"empty set"