$p \rightarrow q$ $\qquad\qquad$ $\sim(p \rightarrow q) \equiv p \wedge \sim q$

28) If $A \subseteq B$ then $A - B = \emptyset$ $\qquad$ $\exists A, B$ s.t. $A \subseteq B \wedge A - B \neq \emptyset$

Proof: Suppose A and B are sets s.t. $A \subseteq B$ but $A - B \neq \emptyset$.

Find $x \in A - B$. $\qquad$ $\forall x,\ x \in A \rightarrow x \in B$

Then $x \in A \wedge x \notin B$. (def of set diff)

$\therefore x \in B$ $\qquad$ $(A \subseteq B)$

$\Rightarrow\Leftarrow$

$\therefore \forall$ sets A, B, $A \subseteq B \rightarrow A - B = \emptyset$.

$B - A = B \cap A^c$

26) $A \cup (B - A) = A \cup B$

Proof: $A \cup (B - A) = A \cup (B \cap A^c)$ $\qquad$ $p \vee \sim p$

$= (A \cup B) \cap (A \cup A^c)$ (dist. law for $\cup$)

$= (A \cup B) \cap U$

$= A \cup B$

Thm: For all sets A, B $\quad A - B = A \cap B^c$

Proof: 1) $A - B \subseteq A \cap B^c$

Let $x \in A - B$

Then $x \in A \wedge x \notin B$

and also $x \in B^c$

$\therefore x \in A \cap B^c$

$\therefore A - B \subseteq A \cap B^c$

2) $A \cap B^c \subseteq A - B$

Let $x \in A \cap B^c$

Then $x \in A \wedge x \in B^c$

and also $x \notin B$

$\therefore x \in A - B$

$\therefore A \cap B^c \subseteq A - B$

$\therefore A \cap B^c = A - B$

Thm: $\forall n \in \mathbb{Z}^{\text{nonneg}}$, for any set with n elements, the powerset has 2^n elements.

Proof: Base case $(n=0)$: Let A have 0 elts. Then $A = \emptyset$.
$\mathcal{P}(\emptyset) = \{\emptyset\}$, which has 1 elt.

$X \in \mathcal{P}(A)$
$\Updownarrow$
$X \subseteq A$

Inductive step: Assume $k \geq 0$ and any set with k elts has a 2^k-elt power set.

$\emptyset \subseteq \emptyset$

[want all sets with $k+1$ elts have 2^{k+1}-elt $\mathcal{P}$]

$X = \{1,2,3,4\}$, $x = 3$

Suppose X is a $(k+1)$ elt set.

Find $x \in X$ (axiom of choice; $X \neq \emptyset$)

Consider $X - \{x\}$, which is a k-elt set

$\therefore \mathcal{P}(X-\{x\})$ has 2^k elts (ind hyp)

Take any $\underline{A} \in \mathcal{P}(X-\{x\})$.

Consider $\underline{A \cup \{x\}}$. $A \cup \{x\} \in \mathcal{P}(X)$

There are as many A's as $\underline{(A \cup \{x\})}$s, so total is $2 \cdot 2^k = 2^{k+1}$.

Every $B \in \mathcal{P}(X)$ is either an A or an $A \cup \{x\}$.

$\mathcal{P}(X-\{x\})$	
$\emptyset$	$\{3\}$
$\{1\}$ $\longrightarrow$	$\{1,3\}$
$\{2\}$ $\longrightarrow$	$\{2,3\}$
$\{4\}$	$\{3,4\}$
$\{1,2\}$	$\{1,2,3\}$
$\{1,4\}$	$\{1,3,4\}$
$\{2,4\}$	$\{2,3,4\}$
$\{1,2,4\}$	$\{1,2,3,4\}$