13) For all sets \( A, B \), \( A \subseteq B \rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B) \)

Proof: Suppose \( A, B \) are sets and \( A \subseteq B \).

- Let \( X \in \mathcal{P}(A) \)
  - Then \( X \subseteq A \) (def \( \mathcal{P} \))
  - \( \therefore X \subseteq B \) (\( \subseteq \) is transitive)

- \( \therefore X \in \mathcal{P}(B) \)
- \( \therefore \mathcal{P}(A) \subseteq \mathcal{P}(B) \)

16) For all sets \( A, B \), \( \mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B) \)

Need: \( \mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B) \)

and \( \mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B) \)

Let \( X \in \mathcal{P}(A \cap B) \)
- Then \( X \subseteq A \cap B \) (def of \( \mathcal{P} \))
- \( \therefore X \in \mathcal{P}(A) \) n \( X \in \mathcal{P}(B) \)

random: can’t predict what will happen

sample space: set of all possible outcomes

event: subset of sample space

if \( S \) is a finite sample space of equally likely outcomes, and \( E \subseteq S \) then

\[
P(E) = \frac{\# \text{ outcomes in } E}{\# \text{ outcomes in } S} = \frac{N(E)}{N(S)}
\]

Ex: flip 2 coins \( S = \{0 \text{ heads, 1 head, 2 heads} \} \)

\( E = \{1 \text{ head} \} \)

\[
P(1 \text{ head}) = \frac{N(E)}{N(S)} = \frac{2}{4} = \frac{1}{2}
\]

\( S = \{TT, TH, HT, HH\} \)

\( E = \{TH, HT\} \)

\[
P(1 \text{ head}) = \frac{N(E)}{N(S)} = \frac{2}{4} = \frac{1}{2}
\]
Ex: roll 2 dice \( S = \{11, 12, 13, \ldots, 16, 21, \ldots, 26, 31, \ldots \} \)

\[
P(\text{total is 7}) = \frac{N(16, 25, 34, 43, 52, 61)}{N(36)} = \frac{6}{36} = \frac{1}{6}
\]

\[
P(\text{total is} \leq 6) = \frac{N(11, 12, 13, 14, 15, 21, 22, 23, 24, 31, 32, 33, 41, 42, 51, 52)}{N(36)} = \frac{15}{36} = \frac{5}{12}
\]

Ex: \( P(\text{draw a black face}) = \frac{\# \text{ of black face cards}}{\# \text{ cards}} = \frac{6}{52} \)

Multiplication rule: if an operation has \( k \) steps w/ 

- \( n_1 \) outcomes for 1st step 
- \( n_2 \) outcomes for 2nd step 

\[\vdots\]
- \( n_k \) outcomes for \( k \)th step

Then total \# outcomes for operation is 

\[n_1 \cdot n_2 \cdot \ldots \cdot n_k\]

MD license plates: 

1) pick 1st letter \( 26 \)
2) " 2nd " \( 26 \)
3) " 3rd " \( 26 \)
4) pick 1st digit \( 10 \)
5) " 2nd " \( 10 \)
6) " 3rd " \( 10 \)

\( 26^3 \cdot 10^3 \approx 17.5 \text{ million} \)
\[ A = \{1, 2, 3, 3\} \quad B = \{u, v, 3\} \quad C = \{m, n, 3\} \]

Elements of \( A \times B \times C \):
1. Pick 1st comp \( (\text{el} A) \) \( \#3 \)
2. Pick el \( B \) \( \#2 \)
3. Pick el \( C \) \( \#2 \)

\( 3 \times 2 \times 2 = 12 \)

4-digit PIN w/ no repeated digits:
1. Pick 1st digit \( 10 \)
2. " 2nd " \( 9 \)
3. " 3rd " \( 8 \)
4. " 4th " \( 7 \)

10 \times 9 \times 8 \times 7 = 5040

Choose LF, CF, RF from Matos, Surhoff, Newhan, Gibbons

Where CF must be M or N and Gibbons must play RF

1. Choose LF \( \geq \) choose M, N choose neither
2. Choose RF \( \leq \) \( \leq \)
3. Choose CF \( \geq \) 0 choices \( \geq \) 2 choices
   no mult rule!

1. Choose CF \( \geq \) 2
2. Choose CF \( \leq \) 2
3. Choose RF \( \geq \) 2

8 total
1) choose LF
2) choose RF
3) choose CF