4-digit PIN, no repeats: 1) pick 1st digit 10 n
2) " 2nd " 9 n-1
3) " 3rd " 8 n-2
4) " 4th " 7 n-3

Total # such PINs = 10 * 9 * 8 * 7 = 5040

Permutations: ordering of a set of objects
Ex: {a, b, c} permutations: abc, acb, bac, bca, cba, cab

Building a permutation of n items: 1) pick 1st item in perm n
2) pick 2nd item n-1
# permutations of n items
= n * (n-1) * (n-2) * ... * 1
= n!

# of permutations of QUIET = 5!

r-permutations: permutation of an r-element subset
# r-permutations of n elements = P(n, r)
= n(n-1)(n-2)...(n-r+1)
= \frac{n!}{(n-r)!}

# 4-digit PINs w/no repeats = P(10, 4) = \frac{10!}{6!} = 5040

# flops in Texas Hold’Em = P(52, 5)
(considering order as important)
= \frac{52!}{(52-5)!} = \frac{52!}{47!} 
= 52 * 51 * 50 * 49 * 48
If $A_1, \ldots, A_k$ are mutually disjoint, then

$$N(A_1 \cup \ldots \cup A_k) = N(A_1) + \ldots + N(A_k)$$

Ex: # 3 digit integers divisible by 5

$\rightarrow$ # 3 digit integers ending in 0

$\rightarrow$ # 3 digit integers ending in 5

$\rightarrow$ 100, 105, 110, \ldots, 995

$\rightarrow$ # 3 digit integers ending in 0 + # 3 digit integers ending in 5

$\rightarrow$ 90 + 90

= 180

205, 215, 225, \ldots, 1995

$\rightarrow$ Length of this list

= 199 - 19

= 180

$n, n+1, \ldots, m$

$\rightarrow$ $m - (n-1)$

$\rightarrow$ $m - n + 1$

If $A$ finite, $B \subseteq A$ then $N(A - B) = N(A) - N(B)$

let $A = U$

$N(U - B) = N(U) - N(B)$

$B^c \subseteq U$

$N(B) = N(U) - N(B^c)$

Ex: # 4 digit PINs that have a repeated digit

1) pick 1st digit 10

2) pick 2nd digit 10

3) pick 3rd digit 10

4) pick 4th digit 7

= # 4-digit PINs - # 4-digit PINs w/o repeat

= $10^4 - P(10, 4)$

= $10^4 - 5040 = 4960$
Inclusion/exclusion:

\[ N(A \cup B) = N(A) + N(B) - N(A \cap B) \]
\[ N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C) \]

Ex: # of multiples of 3 or 5 in 1...1000

\[
= \text{# mults 3 in 1...1000} \quad 1, 3, 2 \times 3, \ldots, 333 \times 3 \\
+ \text{# mults 5 in 1...1000} \quad 1, 5, 2 \times 5, \ldots, 200 \times 5 \\
- \text{# mults of 3 and 5 in 1...1000} \quad 1, 15, 2 \times 15, \ldots, 66 \times 15 \\
= 333 + 200 - 66 = 467
\]

IP addresses: \( w, x, y, z \)

1) pick \( w \) \( 256 \)
2) \( x \) \( 256 \)
3) \( y \) \( 256 \)
4) \( z \) \( 256 \)

Total # IP addresses =

\[ 256^4 = 2^{3 \times 2} \]

\[ = 4 \text{ billion} \]