A function from $X$ to $Y$ ($f : X \rightarrow Y$) is a subset of $X \times Y$ s.t. every element of $X$ has exactly one element of $Y$ s.t. $(x, y) \in f$

$\text{range of } f : X \rightarrow Y = \{ y \in Y \mid \exists x \in X \text{ s.t. } f(x) = y \}$

Define $g$ by $g(y)$

$\text{range of } g \text{ is } \{ m \}$

$g^{-1}(m) = \{ 1, 2, 3 \}$

$g^{-1}(m) = \emptyset$

Inverse image of $y \in Y = f^{-1}(y) = \{ x \in X \mid f(x) = y \}$

Identity function $i_X : X \rightarrow X$ is defined by $i_X(x) = x$

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 6x + 2$

Let $g : \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $g(x) = x^2$

Let $h : \mathbb{Z} \times \mathbb{Z}^+ \rightarrow \mathbb{Q}$ be defined by $h(x, y) = \frac{x}{y}$ not well-defined ($y = 0$)

$f : X \rightarrow Y$ is said to be one-to-one (injective) iff

$\forall x_1, x_2 \in X, x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$h$ is not 1-1 since $(8, 4) \neq (4, 2)$ but $h(8, 4) = \frac{8}{4} = h(4, 2)$

$g$ is not 1-1 since $-1 \neq 1$ but $g(-1) = g(1) = 1$

$f$ is 1-1

Proof: Suppose $x_1, x_2 \in X$ and $f(x_1) = f(x_2)$. [want $x_1 = x_2$]

Then $6x_1 + 2 = 6x_2 + 2$

and $6x_1 = 6x_2$

and $x_1 = x_2$

$\therefore f$ is 1-1