§ 7.1 41) \[ f(A) = \{ x \mid f(x) \in A \} \]

**Ex:** \[ f(x) = 2x \quad f(\{1, 4, 5\}) = \{2, 8, 10\} \]

\[ f(A \cup B) = f(A) \cup f(B) \]

**Proof:** Let \( y \in f(A \cup B) \)

Then \( y = f(x) \) for some \( x \in A \cup B \).

2 cases:
1. \( x \in A \) and so \( f(x) \in f(A) \)
   and \( y \in f(A) \cup f(B) \)
2. \( x \in B \) and so \( f(x) \in f(B) \)
   so \( y \in f(B) \cup f(A) \)

In both cases \( y \in f(A) \cup f(B) \)

\[ f(A \cup B) \subseteq f(A) \cup f(B) \]

Let \( y \in f(A) \cup f(B) \) \[\text{[want } y \in f(A \cup B)\]}

\[ \text{not onto} \]

§ 7.3 33) If \( f, g \) both onto, is \( f \circ g \) necessarily onto?

\[ f(x) = x - 1 \quad (f \circ g)(x) = -1 \]

\[ g(x) = -x \]

\[ f + g(x) = \text{not onto} \]

§ 7.4 17) If \( f : X \rightarrow Y \), \( g : Y \rightarrow Z \) are both onto, must both \( f, g \) be onto?

\( (g \circ f)(x) = \text{onto} \) if \( f \) wasn't
§7.5 15) Can we write (infinite) list of all strings of 0's & 1's? (Given any string, be able to figure out where it is in the list)

\[ \epsilon, \epsilon, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, \ldots \]

0-length strings \( \epsilon \)

length-1 \( \ldots \)

length-2 \( \)

length-3 \( \ldots \)

1010 would be at pos \( 1 + 2 + 4 + 8 + 10 = 25 \)

the fun that gives pos of any string is a bijection from \( \{0,1\}^* \to \mathbb{N} \)

phone-book ordering: \( \epsilon, 0, 00, 000, 0000, \ldots \ldots \) bad list!

Is \( \mathbb{N} \times \mathbb{N} \) countable? (Can we list them?)

\[(0,0), (0,1), (0,2), (0,3), \ldots \]

\[(1,0), (1,1), (1,2), \ldots \]

\[(2,0), (2,1), \ldots \]

\[
(3,0), \ldots
\]

\[
(0,0), (1,0), (0,1), (2,0), (1,1), (0,2), (3,0), \ldots \ldots
\]