

CS 295: Final Exam Practice Problems

Problem 0: Review old quizzes, homeworks, and exams.

Problem 1: Write negations of the following sentences.

- (a) Rafael Palmerio uses steroids but he is not a good hitter.
- (b) If Burt drives a Camaro then Candy likes Burt.
- (c) Anyone who drives a BMW is rich.
- (d) All dairy farmers have a machine that can milk any cow.

Problem 2: Show that the following argument form is valid.

$$\begin{array}{l} p \rightarrow q \\ \neg p \rightarrow r \\ \hline \therefore q \vee r \end{array}$$

Problem 3: Design a circuit with four inputs that outputs 1 if and only if at least three of its inputs are 1.

Problem 4: Convert the decimal numbers in the following list to 8-bit two's complement and convert the binary numbers to decimal. When converting binary numbers, interpret the number first as an unsigned number and then as a signed number encoded using 8-bit two's complement.

- 100_{10}
- -53_{10}
- 01001110_2
- 10101010_2

Problem 5: Let $L(x, y)$ be the predicate “ x lives in y ”. Let $S(x, y)$ be the predicate “ x shops at y ”. Let the domains of the variables and the truth of the predicates for particular values be as indicated by the tables given below. Use P for the set of people, C for the set of cities (ignoring the fact that Columbia and Timonium are not cities), and S for the set of stores.

	<i>L</i>			<i>S</i>		
	Columbia	Baltimore	Timonium	Giant	Wegman's	SuperFresh
Eastman	x			x		
Glenn	x			x	x	
Lawrie	x			x		x
Binkley			x			x
Hall		x		x		

Write each of the following English statements symbolically and determine whether they are true or false. Explain your answers briefly.

- (a) Eastman shops at Giant.
- (b) Everyone who lives in Columbia shops at Giant.
- (c) No one shops at two different stores.
- (d) There is a store whose patrons all live in Columbia.
- (e) There is a place whose residents all shop at the same store.

Problem 6: Prove or disprove: for any integers a , b , c , and d , if $a + b \mid d$ and $a + c \mid d$ then $b + c \mid d$.

Problem 7: Find an integer r such that $0 \leq r < 13$ and $11^8 \equiv r \pmod{13}$.

Problem 8: Prove that $\sqrt{\frac{1}{6}}$ is irrational.

Problem 9: Rewrite each of the following using summation notation.

- (a) $4 + 7 + \cdots + 31$
- (b) $8 + \cdots + n!n^2$
- (c) $-2 + 8 - \cdots + 8n^2$

Problem 10: Find a formula for $\sum_{i=1}^n 4i + 3$. Check your answer by proving it by induction.

Problem 11: Find the smallest k such that any amount of at least k cents can be made with 4-cent and 5-cent stamps. Prove your answer.

Problem 12: Prove that the following code fragment correctly computes $\lfloor \log_2 a \rfloor$.

[Precondition: $A \geq 1.0, l = 0, b = A$]

```
while (b >= 2.0)
  b = b / 2.0          // note that A,b are reals, not integers
  l = l + 1
end while
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[Postcondition: $l = \lfloor \log_2 a \rfloor$.]

(Hint: part of your invariant can be $b \cdot 2^l = a$; review properties of floors and ceilings.)

Problem 13: Prove that for any sets A and B , $A \cup (A \cap B) = A$ using no properties of sets other than the definitions of the set operations.

Problem 14: Prove that, for any sets A , B , C , and D , if $C \subseteq A - B$ and $D \subseteq B - A$ then C and D are disjoint.

Problem 15: Prove or disprove: for any sets A and B , $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$.

Problem 16: A state issues license plates in two formats: either two letters followed by three numbers, or 5 numbers with a letter somewhere between them (but not at the ends). How many different license plates can the state issue?

Problem 17: Imagine a game like poker but played with 4 card hands. Which should be a better hand: one pair or “melting pot”, which is four cards all of different ranks and different suits.

Problem 18: 50 people were surveyed about what TV shows they watch. 21 reported that they watch The Amazing Race, 25 watch Veronica Mars, and 11 watch Arrested Development. 3 watch AD and TAR, 9 watch TAR and VM, 3 watch VM and AD and 1 watches all 3. Fill in the Venn diagram that shows how many people watch each possible combination of shows.

Problem 19: 12 people are to be seated for a jury. The jury pool consists of 20 people: 7 college students, 10 retirees, and 3 working professionals. How many possible juries are there? How many have an equal number from each group? How many include no retirees? How many include at most 3 retirees? How many include all the college students? How many include more working professionals than college students?

Problem 20: The game Can't Stop is played with 4 indistinguishable 6-sided dice. How many distinct outcomes are there of rolling the dice?

Problem 21: Define $f : \mathbf{R} \rightarrow \mathbf{R}$ by $f(x) = (x - 1)(x + 2)(x + 3)$. Is f 1-1? Is f onto? Explain your answers.

Problem 22: Prove that the set of squares of integers $\{0, 1, 4, \dots\}$ is countably infinite by finding a bijection between that set and \mathbf{Z}^+ .

Problem 23: Define $f : \mathbf{R}^{nonneg} \rightarrow \mathbf{R}$ by $f(x) = x^2 + 3x + 7$ and $g : \mathbf{R}^{nonneg} \rightarrow \mathbf{R}$ by $g(x) = 3x^2 - 9x + 100$. Prove that $f(x)$ is $\Theta(g(x))$.