Either Dom doesn't play drums or Dom plays bass.

$\sim p \vee q$

Dom plays drums but not bass $\quad p \wedge \sim q$

Dom doesn't play both drums and bass $\quad \sim(p \wedge q)$

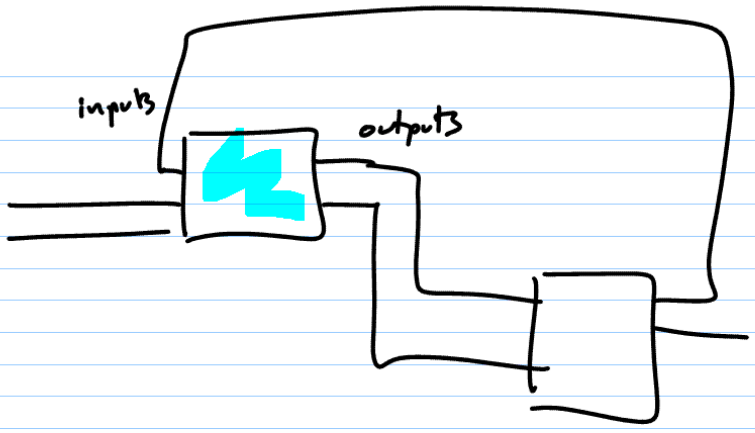
$\sim p \vee \sim q$

It is not the case that Dom plays drums but not bass.

$\sim(p \wedge \sim q) \equiv \sim p \vee q$

If Dom plays drums then he plays bass.

$p \rightarrow q \quad \equiv \sim p \vee q$

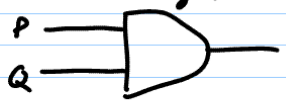


digital circuit: current on wire or not (on / off)

black boxes: can describe behavior based on what outputs are for different inputs

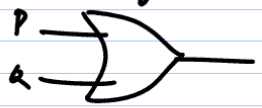
basic black boxes

and gate



P	Q	output
1	1	1
1	0	0
0	1	0
0	0	0

or gate



P	Q	output
1	1	1
1	0	1
0	1	1
0	0	0

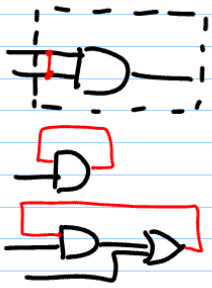
not gate

P	output
1	0
0	1

I/O tables

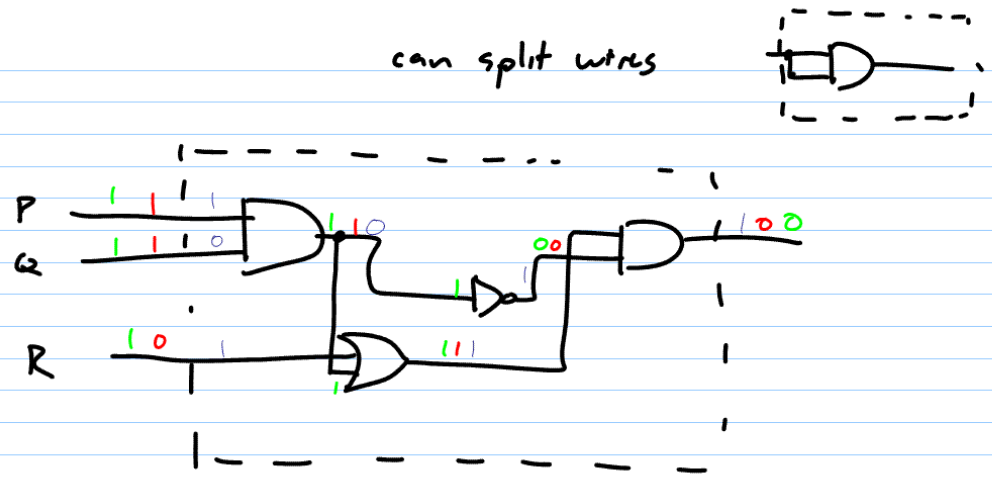
combinational circuit: can't combine inputs

can't connect output back to input

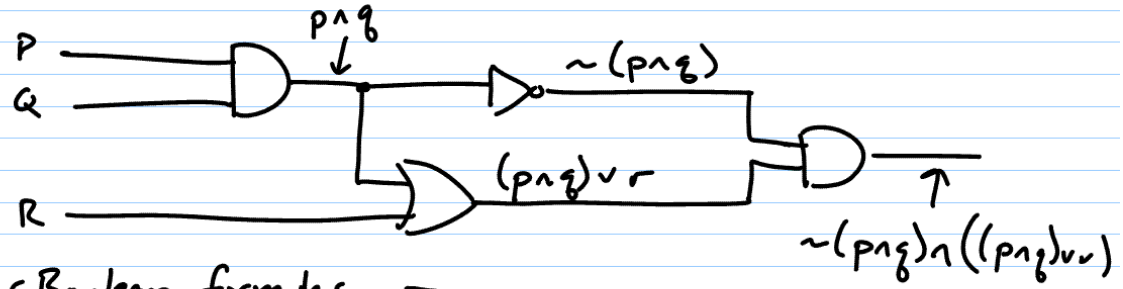


can have output of 1 gate as input of another

can split wires

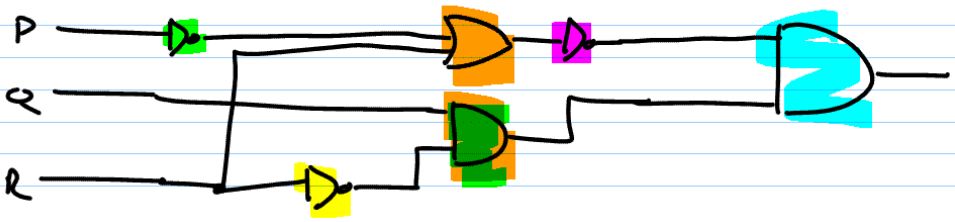


P	Q	R	output
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0



Boolean formulas
↓
combinational circuits
↓
I/O table

$$(\sim(\sim p \vee r)) \wedge (q \wedge \sim r)$$



I/O → circuit

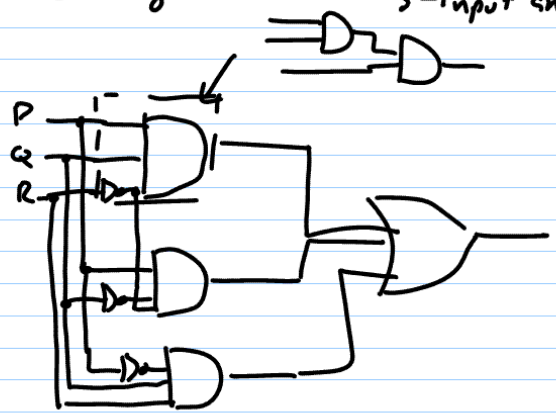
1) pick rows where output on (1)

2) build recognizer for each row

↳ output is on for (recognizes) exactly one combination of inputs

3) or outputs of recognizers together

P	Q	R	output
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0



binary numbers allow integers to be encoded on wires

↳ use digits 0/1 in place value system

with place values = powers of 2

$$\underset{100}{3}\ \underset{10}{7}\ \underset{1}{1} = 3\cdot 100 + 7\cdot 10 + 1\cdot 1$$
$$= 3\cdot 10^2 + 7\cdot 10^1 + 1\cdot 10^0$$

$$\underset{128}{1}\ \underset{64}{0}\ \underset{32}{1}\ \underset{16}{1}\ \underset{8}{0}\ \underset{4}{1}\ \underset{2}{0}\ \underset{1}{1} = 1\cdot 1 + 0\cdot 2 + 1\cdot 4 + 0\cdot 8 + 1\cdot 16 + 1\cdot 32 + 0\cdot 64 + 1\cdot 128$$

$$= 181$$

$$\begin{array}{r} {}^{1}7 \\ +\ 3\,6 \\ \hline 1 \end{array}$$

$7+4 = 11_2$

$$\begin{array}{rr} & 0\ 1\ 0\ 0\ 1\ 1\ 1 \\ & 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1 \\ + & 0\ 0\ 1\ 0\ 0\ 1\ 1\ 1 \\ \hline & 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0 \end{array} \qquad \begin{array}{r} 181 \\ 39 \\ \hline 220 \end{array}$$

$1+1 = 2 = 10_2$

P	Q	carry	sum
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

I/O table for half-adder