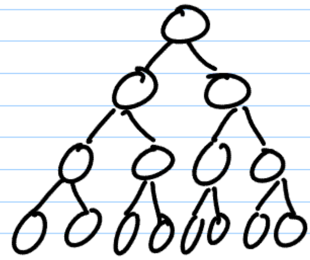

1, 2, 4, 8, 16, ...

Define sequence a by

$a_i = 2^i$ for $i \geq 0$

OR

$a_0 = 1$ and $a_i = 2 \cdot a_{i-1}$ for $i \geq 1$

$f_0 = 0$, $f_1 = 1$, $f_i = f_{i-1} + f_{i-2}$ for $i \geq 2$

f_3 f_4

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... Fibonacci sequence

$f_2 = f_1 + f_0$ $f_5 = f_4 + f_3$

\# nodes in 5 level tree $a_0 + a_1 + a_2 + a_3 + a_4$

\# nodes in 36 level tree $a_0 + a_1 + a_2 + \cdots + a_{35}$

$= \sum_{i=0}^{35} a_i$

$$\sum_{i=1}^{5}(i^2+1) = 2+5+10+17+26$$

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\frac{7}{8} = \sum_{i=1}^{7}\frac{i}{i+1}$$

$a_1 \quad a_2 \quad a_3 \quad - - - - \quad a_7$

$b_0 \quad b_1 \quad b_2 \quad - -$

$$a_i = \frac{i}{i+1} \qquad = \sum_{j=0}^{6}\frac{i+1}{i+2}$$

$$b_i = \frac{i+1}{i+2}$$

$1\cdot 2 \quad 2\cdot 3 \quad 3\cdot 4 \quad 4\cdot 5$

$$2-6+12-20 = \sum_{i=1}^{4} i(i+1)\cdot(-1)^{i+1}$$

$c_1 \quad c_2 \quad c_3 \quad c_4$

$$= 1\cdot 2 + -1\cdot 6 + 1\cdot 12 + -1\cdot 20$$

$$c_i = i(i+1)\cdot(-1)^{i+1} \qquad = 2-6+12-20$$

Σ properties: $\sum_{k=m}^{n} a_k = \sum_{k=m}^{n-1} a_k + a_n$

$$\sum_{k=m}^{i} a_k + \sum_{k=i+1}^{n} a_k = \sum_{k=m}^{n} a_k$$

$a_0 + a_1 + a_2 + b_0 + b_1 + b_2$
$= (a_0 + b_0) + (a_1 + b_1) + (a_2 + b_2)$

$$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$$

$c \cdot a_0 + c \cdot a_1 + c \cdot a_2$
$= c(a_0 + a_1 + a_2)$

$$\sum_{k=m}^{n} c \cdot a_k = c \cdot \sum_{k=m}^{n} a_k$$

Π notation for products: $\prod_{k=m}^{n} a_k = a_m \cdot a_{m+1} \cdot \dots \cdot a_n$

$$n! = \prod_{i=1}^{n} i$$

$7! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$

$0! = 1$ (empty product is defined to be 1)

$$\sum_{k=0}^{6} \frac{1}{k+1} + \sum_{j=1}^{7} \frac{1}{j^2} = \sum_{i=1}^{7} \frac{1}{i} + \sum_{j=1}^{7} \frac{1}{j^2}$$

$$= \sum_{j=1}^{7} \frac{1}{j} + \sum_{j=1}^{7} \frac{1}{j^2}$$

$$= \sum_{j=1}^{7} \left(\frac{1}{j} + \frac{1}{j^2} \right)$$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$$

$c_1 \quad c_2 \quad c_3 \; - - - - \quad c_7$

$$c_i = \frac{1}{i}$$

$$\sum_{i=1}^{7} \frac{1}{i}$$

$\Rightarrow k = j - 1$ $\qquad \displaystyle\sum_{j=1}^{7} \frac{1}{(j-1)+1} = \sum_{j=1}^{7} \frac{1}{j}$

$j = k+1$

Define sequence a by $a_0 = 7$ and $a_k = 3 \cdot a_{k-1} + 4$ for $k \geq 1$

$$\left(\underset{a_0}{7}, \underset{a_1}{25}, \underset{a_2}{79}, 241, \ldots \right)$$

Wow! each a_k is 1 more than a mult of 3

$\forall k \geq 0 \quad a_k \equiv 1 \pmod 3$

☆ $a_0 \equiv 1 \pmod 3$

$a_0 \equiv 1 \rightarrow a_1 \equiv 1 \pmod 3$

k=1 $\therefore a_1 \equiv 1 \pmod 3$ ☆ $\forall k \in \mathbb{Z}, k \geq 0 \wedge a_k \equiv 1 \pmod 3 \rightarrow a_{k+1} \equiv 1 \pmod 3$

$(\cancel{k \geq 0} \wedge)$ T $a_1 \equiv 1 \rightarrow a_2 \equiv 1 \pmod 3$

k=2 $\therefore a_2 \equiv 1 \pmod 3$

$a_2 \equiv 1 \rightarrow a_3 \equiv 1 \pmod 3$

k=3 $\therefore a_3 \equiv 1 \pmod 3$ $\forall k \in \mathbb{Z}, k \geq 0 \rightarrow a_k \equiv 1 \pmod 3$

$\vdots$

Principle of Mathematical Induction

Let $P(n)$ be predicate with domain $\mathbb{Z}$

If you know 1) $P(a)$ is T for some $a \in \mathbb{Z}$

2) $\forall k \in \mathbb{Z}, k \geq a \wedge P(k) \rightarrow P(k+1)$

Then can conclude $\forall k \in \mathbb{Z}, k \geq a \rightarrow P(k)$