Computations performed by \( P_1, \ldots, P_n \)

\( Z \) computes to use pieces \( P_1, \ldots, P_n \) independent

\( P_i \) takes \( t_i \) to complete

Final task \( Q \) run after all \( P_1, \ldots, P_n \) finish

Goal: finish task \( Q \) as soon as possible

\[ t_{\text{times}}: \begin{array}{cccc}
    9 & 7 & 6 & 4 \\
    1 & 2 & 3 & 5 \\
  \end{array} \]

\[ \text{CPU 1} \quad \text{CPU 2} \]

can start \( Q \) at time \( 28 \)

better split: \( 9, 7, 6, 4 \) / \( 1, 2, 3, 5 \)

\[ \begin{array}{c}
    \text{19} \\
    \text{18} \\
  \end{array} \]

start \( Q \) at time \( 19 \)
How to find best split?

For every possible split, compute max time for split; keeping track of maxes.

For $n=8$, $k=4$, $(\binom{8}{4})$ split to examine.

$n=8$, $k$ = anything $2^8$

1) Pick chunk $i$ on CPU for $2^2$

2) $\frac{2}{3}$

3) $\frac{3}{2}$

$n$'s $\frac{2}{n}$

$N(\emptyset(A)) = 2^{\mu(A)}$

for chunk $i$ $\emptyset A$.
12 chunks, want to put 5 chunks on CPU 1

How many possible ways are there to do that? \( \binom{12}{5} \)

Suppose that chunk 1 and chunk 2 must be on different CPUs

splits w/ 1 & 2 on different CPUs

= 

\[ \begin{align*}
\text{Splits w/ 1 on CPU 1, 2 on CPU 2} & \quad \text{1) put chunk 1 on CPU 1, 2) pick 4 more, but not chunk 2} \\
\text{Splits w/ 1 on CPU 1, 2 on CPU 2} & \quad \text{1) put chunk 2 on CPU 1, 2) pick 4 more, but not chunk 1}
\end{align*} \]

\[ \binom{10}{4} + \binom{10}{4} = \binom{10}{4} \]

Total splits w/ 1 & 2 on different CPUs = \( \binom{10}{4} + \binom{10}{4} = \binom{10}{4} \)
10 students (7 men, 3 women) - form a 5-person team

Ben hates Cathy - can't put both on team

How many ways to make 5-person teams

\[
\binom{8}{4} + \binom{8}{4} + \binom{8}{5}
\]

\[\begin{align*}
\text{choose B,} & \quad \text{choose C,} & \quad \text{don't choose} \\
\text{can't choose} & \quad \text{not B} & \quad \text{either}
\end{align*}\]

How many terms have 3 men, 2 women (Ben likes Cathy now)

1) choose women \(\binom{3}{2}\)

2) choose men \(\binom{7}{3}\)

\[\frac{\binom{3}{2}\binom{7}{3}}{\binom{5}{2}}\]
How many teams have ≥ 1 woman? Total teams = \(\binom{5}{5}\), no women = \(\binom{2}{5}\)

\[
\left(\frac{\binom{5}{5}}{\binom{2}{5}}\right) - \left(\frac{\binom{2}{5}}{\binom{2}{5}}\right)
\]

Teams w/1 woman + teams w/2 women + teams w/3 women

\[
\binom{3}{5} + \binom{3}{2} + \binom{3}{3}
\]

50 people polled

- 26 liked O's
- 4 liked O's & Ravens
- 1 liked O's, Ravens, Redskins
- 45 liked ≥ 1 team
- 18 like Redskins
- 5 like Redskins & Ravens
- 16 like Ravens

\[
x + y + 4 = 26
\]
\[
y + z + 5 = 16
\]
\[
x + y + z = 29
\]

\[
x = 16 \rightarrow y = 6 \rightarrow z = 7
\]
# of different outcomes when rolling 5 6-sided dice

\[ = \binom{10}{6} \]

in general # outcomes when rolling \( r \) \( n \)-sided dice

\[ = \binom{n+r-1}{r} \]

\# multisets of size \( r \) chosen from \( n \) things

\[ \rightarrow \text{sets with repetition (order still doesn't matter)} \]

\[[1,1,2] \neq [1,2] \]

\[[1,1,2] = [1,2,1] \]