Size-$r$ subset of $\{1, \ldots, n\}$

Vending Machine

buy 11 cans of juice (8 kinds)

do optimize nutrition

brute force

for each collection of 11 cans of juice,
compute nutrition
keeping track of best so far

how many collections do we take over

# of size-11 multisets of an 8-set set $r$

\[ \binom{n+r-1}{r} \]

1) buy one can 6 apple orange
2) buy another can 3 apple apple
11) buy 11th can 8 other apple

8''
double counts
one-to-one
(does $x \neq x_2$ guarantee $x_1 \neq x_2$)
$y = x^2$

$y = \sqrt[3]{x}$

undefined for negatives

$y = \sqrt{x}$

$(-2, 4)$
$(2, 4)$

not 1-1
$f(1) = f(-1)$

not onto since no $x$ makes $y = x^2$ makes $f(x) = -1$

$x^2 = -1$

$(-2, 4)$
$(2, 4)$

2 possible $y$ values for $x = 0$
A function from $X$ to $Y$ ($f: X \to Y$) is a subset of $X \times Y$ (set of ordered pairs $(x, f(x))$) if every element $x$ of $X$ has exactly one element of $Y$ s.t. $(x, y) \in f$.

\[ f = \{ (a, 1), (b, 1), (c, 3), (d, 2) \} \]

\[ f(a) = 1 \quad f(c) = 3 \]
\[ f(b) = 1 \quad f(d) = 2 \]

$f: X \to Y$ is one-to-one (injective) iff (for $f: \mathbb{R} \to \mathbb{R}$, horizontal lines cut curve at most once)

\[ \forall x_1, x_2 \in X, f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \]

\[ x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \]

$f: X \to Y$ is onto (surjective) iff (for $f: \mathbb{R} \to \mathbb{R}$, horizontal lines cut curve at least once)

\[ \forall y \in Y, \exists x \in X \text{ s.t. } f(x) = y \]
\[ y = 2^x \]

Let \( f : \mathbb{R} \to \mathbb{R} \) be \( f(x) = 2^x \). 

\( f \) is not onto: 

Cannot solve \( 2^x = -100 \)

Let \( g : \mathbb{R} \to \mathbb{R^+} \) be \( g(x) = 2^x \).

\( g \) is onto:

\[ \forall y \in \mathbb{R^+} \exists x \in \mathbb{R} \text{ s.t. } y = 2^x \]

Namely \( x = \log_2 y \)

\[ f : \mathbb{R} \to \mathbb{R}, \ f(x) = x^2 \text{ is not } 1-1 \]

\[ g : \mathbb{R^+} \to \mathbb{R}, \ g(x) = x^2 \text{ is } 1-1 \]