1b) 2 9-sided dice

$$P(\text{total is even}) = \frac{N(\text{ways to get even tot})}{9^2} = \frac{41}{81}$$

\# ways to get even, even   \# ways to roll odd odd

$4 \cdot 4$ $5 \cdot 5$

$16 + 25$

41

2a) E _ _ _ _ _ _ _   order 7 cities in remaining 7 pos

$7!$

b) $\underbrace{\_\ \_\ \_\ \_}_{\text{order } A,B,C,D \text{ in } 1^{st} \text{ 4 pos}}\ \underbrace{\_\ \_\ \_\ \_}_{\text{order } E,F,G,H \text{ in } 2^{nd} \text{ 4 pos}}$

$4!$ $4!$

$(4!)^2$

c) A B _ _ _ _ _ _ $6!$

_ B C _ _ _ _ _ $+\,6!$

A B C _ _ _ _ _ $-\,5!$

3a) 4 of a kind :

1) pick rank 13
2) pick 4 cards $\binom{6}{4}$
3) pick 5^{th} card 48

$$13 \cdot \binom{6}{4} \cdot 48$$

b) same color :

1) pick color 2
2) pick 5 cards that color $\binom{28}{5}$

$$2 \cdot \binom{28}{5}$$

Pigeonhole principle : X, Y finite sets , $N(X) > N(Y)$, $f: X \to Y \to f$ not 1-1

also X, Y finite sets, $N(X) < N(Y)$, $f: X \to Y \to f$ not onto

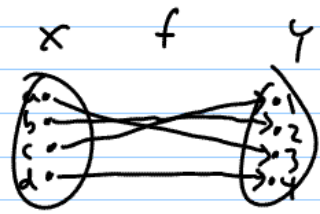
together : X, Y finite sets , $N(X) \neq N(Y)$, $f: X \to Y \to f$ not a bijection

if you've got a bijection $f: X \to Y$ then $N(X) = N(Y)$

finite?

Thm: For finite sets X, Y , if $f: X \to Y$ and $N(X) = N(Y)$ then

f is 1-1 if and only if f is onto

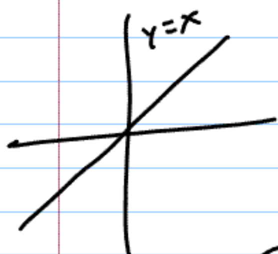


Def: A has same cardinality as B iff $\exists f: A \to B$ s.t. f is a bijection

(cardinality for finite sets is # elts in the set)

Properties: For any sets A, B, C

$i_A(x) = x$



reflexive 1) A has same cardinality as A ($i_A: A \to A$ is a bijection)

symmetric 2) A has same cardinality as B $\to$ B has same cardinality as A (if $f: A \to B$ is bijection, so is f^{-1})

transitive 3) A has same card as B and B has same card as C $\to$ A has same card as C (if $f: A \to B$ is bijection and $g: B \to C$ is bijection then so is $g \circ f$)

equivalence relation

f, g are composable

If $f: X \to Y'$ and $g: Y \to Z$ and $Y' \subseteq Y$ then composition of f with g ($g \circ f$) is defined by $(g \circ f)(x) = g(f(x))$

Thm: If f, g are composable then
if f, g both 1-1 then so is $g \circ f$
if f, g both onto ($Y' = Y$) then so is $g \circ f$

Which is bigger? $\mathbb{Z}$ or $\mathbb{N}$ in one sense, $\mathbb{Z}$, since $\mathbb{N} \subseteq \mathbb{Z}$

But $\mathbb{N}$ has the same cardinality as $\mathbb{Z}$

$= \{2x \mid x \in \mathbb{Z}\}$
$=$ even integers

Thm: $\mathbb{Z}$ has the same cardinality as $2\mathbb{Z}$

Proof: [we need to find $f: \mathbb{Z} \to 2\mathbb{Z}$ s.t. f is a bijection]

let $f(x) = 2x$. Then f is 1-1 : Suppose $f(x_1) = f(x_2)$

then $2x_1 = 2x_2$

$\forall x_1, x_2,\ f(x_1) = f(x_2) \to x_1 = x_2$ and $x_1 = x_2$

Also f is onto: Suppose $y \in 2\mathbb{Z}$. let $x = \frac{y}{2}$. Then

$\forall y \in 2\mathbb{Z}\ \exists x \in \mathbb{Z}$ s.t. $f(x) = y$ $\qquad f(x) = 2 \cdot y/2 = y$

and $x \in \mathbb{Z}$ (since $y \in 2\mathbb{Z}$, $y/2 \in \mathbb{Z}$)

$\therefore \exists f: \mathbb{Z} \to 2\mathbb{Z}$ s.t. f is a bijection

$\therefore$ $\mathbb{Z}$ has same cardinality as $2\mathbb{Z}$

Def: A is countably infinite if and only if A has the same cardinality as $\mathbb{N}$.

Def: A is countable iff A is finite or A is countably infinite

Def: A is uncountable iff A is not countable