To compute $y = a_n x^n + \cdots + a_0$

$$= \sum_{k=0}^{n} a_k \cdot x^k$$

```
  y = 0
  i = n
* while i ≥ 0
        y = a[i] + x * y
        i = i - 1
  wend
```

if
⋮
fi

prove that at *, always have

$$y = \sum_{k=0}^{n-(i+1)} a_{k+i+1} \cdot x^k \qquad \dagger$$

note that last time at * i = -1

this says $y = \sum_{k=0}^{n} a_k \cdot x^k$

Proof : Induction – prove † is T 1st time
– prove if † is T at top of loop, body of loop makes it still T next time at top

Base case: 1st time at $*$, $i = n$, so † is

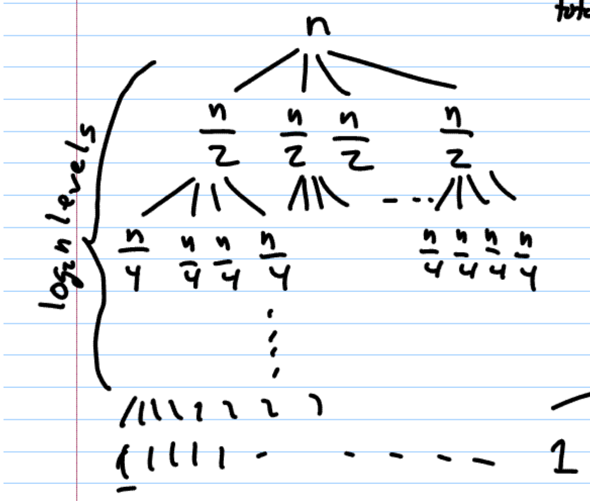
$$y = \sum_{k=0}^{-1} a_{k+n+1} \cdot x^k = 0,$$

$y$ was just set to 0, so † is T

Ind step: Suppose $i \geq 0$ and † is T at $*$
(need to show † is T next time $*$)

p75 4.3-1

a) $T(n) = 4\,T(\frac{n}{2}) + \underline{n}$



total work at level

$n = n$

$4 \cdot \frac{n}{2}$ 2n

$16 \cdot \frac{n}{4}$ 4n

level

1 4 subs of size $\frac{n}{2}$

2 16 sub of size $\frac{n}{4}$

3 64 sub of size $\frac{n}{8}$

k $4^k$ sub of size $\frac{n}{2^k}$

total $\approx (1 + 2 + 4 + 8 + \cdot 2^{\log_2 n})n$

$\approx n \cdot n$

$= \Theta(n^2)$

\# of size-1 sub problems

$= 4^{\log_2 n} = n^2$

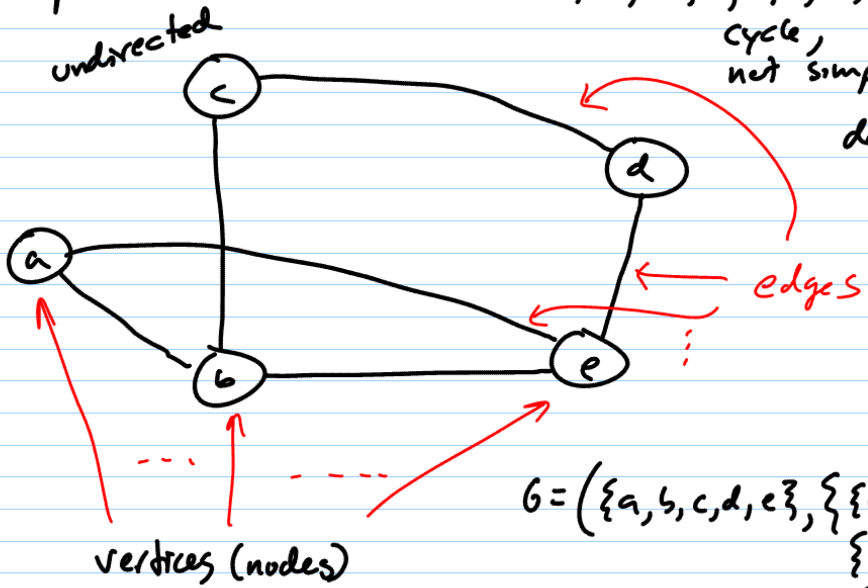
total of 🟩 is $\Theta(n^2)$

Graph

$c, d, \overbrace{e, a, b, e}, a, b, c$

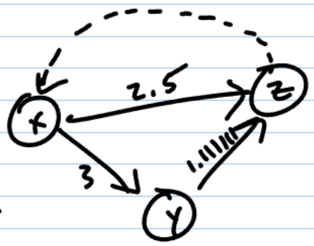
cycle,
not simple cycle

degree(e)
= 3



$G = (\{a, b, c, d, e\}, \{\{a, b\}, \{b, e\}, \{e, d\}, \{a, e\}, \{c, d\}, \{b, c\}\})$

$G = (\underset{\text{set of verts}}{V}, \overset{\text{set of edges}}{E})$ where E contains size-2 subsets of V directed graph, edges have direction $(E \subseteq \underset{\substack{\uparrow \\ \text{from}}}{V} \times \underset{\substack{\uparrow \\ \text{to}}}{V})$



$G = (\{a,b,c\}, \{(x,y),(x,z),$
$(y,z)\})$

degree(x) = 2

outdegree(x) = 2, indegree(x) = 0

weighted graph has $w: E \to \mathbb{R}$

Ex: $w(x,y) = 3$

$w(x,z) = 2.5$

$w(y,z) = 1.111111$

degree of a vertex in undirected graph
= # of edges incident on it

indegree of v in dir graph = # edges in

outdegree " # edges out

path is a sequence of verts $v_0, v_1, \dots, v_k$

s.t. $(v_i, v_{i+1}) \in E$ for $i = 0 \dots k-1$

Ex: x, y, z is a path since $(x,y), (y,z) \in E$

↳ length = 2

length = # of edges

if ∃ path from u to v, write $u \overset{p}{\rightsquigarrow} v$

v is reachable from u (via path p)

simple : no vertex is repeated

cycle : starts where it finishes

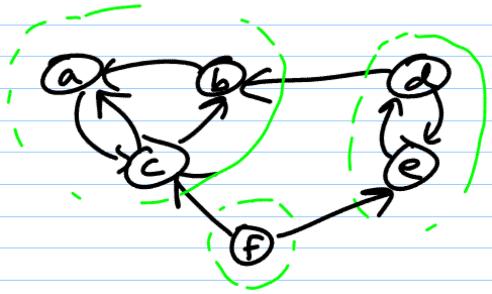
simple cycle: finish is 1st repeated verts

a, b, e, a is simple

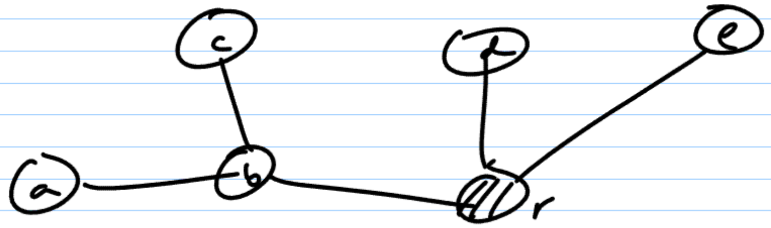
undirected G is
connected: if any vertex is reachable from any other

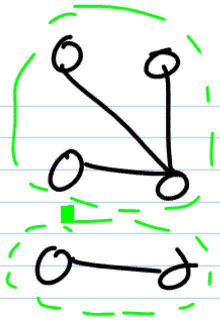
directed graph G is strongly connected:
for any pair $(u,v)$ there is a path $u \rightsquigarrow v$

strongly connected components: maximal subsets of verts
that are strongly connected



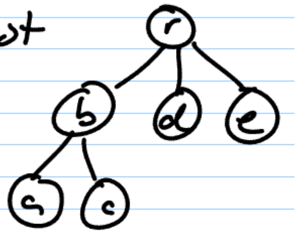
(free) tree: undirected, acyclic graph, connected





rooted tree specifies one vertex as the root

| | |
|---|---|
| parents | children |
| ancestors | descendants |



directed acyclic graph : DAG