0.98  .14  .72  .71  .96  .83  .23  .24  .22  .91

0.1  .14
.2  .3  .23  .24  .22  .23  .24
.3  .4
.4  .5  Sort each bucket
.5
.6  .72  .71
.7  .83
.8  .98  .96  .91
.9  .91  .96  .98

\[ X_{90} = 1 \quad X_{95} = 1 \quad X_{99} = 1 \]
\[ X_{90} = 0 \quad X_{95} = 0 \quad X_{99} = 0 \]

Work done to sort bucket

Running time (expected) for bucket sort: \( O(n) + \sum_{i=0}^{n-1} E[Y_i] \)

Steps 1, 2, 4
\[
\sum_{i=0}^{m-1} E[Y_i] = \sum_{i=0}^{m-1} E[n_i^2]
\]

\(n_i = \# \text{ of things in bucket } i\)

\[X_{ij} = \begin{cases} 1 & \text{if input } j \text{ goes in bucket } i \\ 0 & \text{otherwise} \end{cases}\]

\[E[x \cdot y] = E[x] \cdot E[y]\]

\[n_i = \sum_{j=0}^{m-1} X_{ij}\]

\[E[n_i^2] = \left(\sum_{j=0}^{m-1} X_{ij}\right)^2 = \sum_{j=0}^{m-1} \sum_{k=0}^{m-1} X_{ij} \cdot X_{ik}\]

\[E[n_i^2] = E\left[\sum_{j=0}^{m-1} \sum_{k=0}^{m-1} X_{ij} \cdot X_{ik}\right]\]

\[= \sum_{j=0}^{m-1} \sum_{k=0}^{m-1} E[X_{ij} \cdot X_{ik}]\]

\[1 \cdot P(X_{ij} = 1) + 0 \cdot P(X_{ij} = 0) = \frac{1}{n}\]
\[
\begin{align*}
&= \sum_{i=0}^{a} E[X_{ij}^2] + \sum_{i=0}^{a-1} \sum_{k=0}^{a-1-i} E[X_{ij}] \cdot E[X_{ik}]
\end{align*}
\]

\[1 \cdot P(X_{ij} = 1) = \sum \sum \frac{1}{n^2}
\]

\[+ (1 - P(X_{ij} = 0)) = \sum \sum \frac{1}{n^2} + \frac{n(n-1)}{n^2}
\]

\[= 1 \cdot P(X_{ij} = 1) = \sum \frac{1}{n} + \frac{n(n-1)}{n^2}
\]

\[= \frac{n^2}{n^2} + \frac{n(n-1)}{n^2} = 2 - \frac{1}{n}
\]

\[= O(1) \text{ to sort each bucket}
\]

\[
\text{Total time to sort all buckets } = O(n) \quad \text{(step 4)}
\]

\[
\therefore \text{Total time for all steps is } O(n)
\]
To find max:

\[
\text{max} = a[0] \\
\text{for } i = 1 \text{ to } n-1 \\
\text{if } a[i] > \text{max} \quad O(n) \\
\text{max} = a[i]
\]

To find min and max:

\[
\text{max} = \text{min} = a[0] \\
\text{for } i = 1 \text{ to } n-1 \\
\text{if } a[i] > \text{max} \quad O(n) \\
\text{max} = a[i] \\
\text{else if } a[i] < \text{min} \quad 2n - 2 \text{ comparisons in worst case} \\
\text{min} = a[i] \\
\]

if \((a[0] > a[1])
\]

\[
\text{max} = a[0], \text{ min} = a[1] \\
\text{else } \text{max} = a[1], \text{ min} = a[0]
\]
for \( n \) even

1) pair off \( a_i \)

\[
\frac{n}{2}
\]

2) for each pair, determine larger + smaller

\[
\frac{n}{2} - 1
\]

3) find \( \max \) of larger

\[
\frac{n}{2} - 1
\]

4) find \( \min \) of smaller

\[
\frac{3n}{2} - 2
\]
General select: given n inputs (not necessarily ordered) and i, find ith smallest value

1) sort
2) return a[i]

\( O(n \log n) \)

\[ \text{Randomized-Select}(A, i, p, r) \]

1) \( g = \text{partition}(A, p, r) \)
2) if \( i = g \)
   Expected \( O(n) \)
   return \( A[g] \) comparisons.
else \( i < g \)
   return \( \text{R-S}(A, i, p, g-1) \)
else \( i > g \)
   return \( \text{R-S}(A, i-g+1, r) \)