Select \( (A, i, p, r) \)

1) Break inputs into groups of 5 (or less)

2) Find median of each group (with sorting alg.)
   (this takes \( \leq 7 \) comparisons)

3) Recursively find median of medians (call it \( x \))

4) Partition around \( x \) (find rank of \( x \), call that \( g \))

5) if \( i < g \)
   
   \[ \text{select} (A, i, p, g-1) \]
   
   else if \( i > g \)
   
   \[ \text{select} (A, i-g, g+1, r) \]

   else return \( A[g] \)
$$\lceil \frac{n}{5} \rceil$$ groups total

$$\left\lfloor \frac{1}{2} \left\lceil \frac{n}{5} \right\rceil \right\rfloor - 1$$ groups w/median \( \leq \) of \( x \)

most of those groups have 3 els that are \( \leq x \)
(letters can have as few as 1)

total # of els ≤ x is \[3 \left( \left\lceil \frac{3}{2} \sqrt{n} \right\rceil - 1 \right) - 2\]

\[\geq \frac{3n}{10} - 5\]

\[T(n) = \# \text{ comps done to find rank } i\]

\[\leq 7 \cdot \sqrt{\frac{n}{3}} + T\left( \left\lceil \frac{n}{5} \right\rceil \right) + (n-1) + T\left( \frac{2}{15}n + 5 \right)\]

\[\text{step2 step3 step4 step5}\]

Guess \(T(n) \leq c \cdot n\) for some \(c\)

Base cases : ---- (pick \(c\) big enough to cover base cases)

Ind step : Suppose \(T(i) \leq c \cdot i\) for \(i = 1, \ldots, k-1\)

\[\text{want } T(k) \leq k\]

and \(k > \)
\[ T(k) \leq 7 \cdot \sqrt{\frac{k}{3}} + T\left(\frac{\sqrt{3}k}{2}\right) + (k-1) + T\left(\frac{2}{10}k + 5\right) \]
\[ \leq 7 \cdot \sqrt{\frac{k}{3}} + c \cdot \sqrt{\frac{k}{3}} + (k-1) + c \cdot \left(\frac{2}{10}k + 5\right) \]
\[ \leq 7\left(\frac{k}{3} + 1\right) + c\left(\frac{k}{3} + 1\right) + (k-1) + c\left(\frac{2}{10}k + 5\right) \]
\[ \leq \frac{12}{5}k + 6 + \frac{9}{10}ck + 6c \]

want \( \leq c \cdot k \)

which will work as long as

\[ \frac{12}{5}k + 6 + 6c \leq \frac{1}{10} \cdot c \cdot k \]
\[ 24k + 60 + 60c \leq c \cdot k \]
\[ 24k + 60 \leq (k-60)c \]

\[ \therefore 0(n) \]
\[ \frac{24k + 60}{k-60} \leq c \]
\[ \frac{1}{c-25} \text{ for large enough values of } k \]

have \( c > 25 \) from inductive step

\( c > \frac{97}{39} \) from base case

\( c > \text{ was constant from } \)