v is a descendant of u \iff (d[v], f[v]) \in (d[u], f[u])

**WHITE PATH THM**

v is a descendant of u \iff u \rightarrow v by white path at time d[u]

\[ \rightarrow \setminus \text{ Assume v is a descendant of u.} \]
\[ u \setminus \text{ Then there is a path in the tree } u \rightarrow v; \]
\[ \setminus \text{ Claim that is WHITE:} \]
\[ u \setminus \text{ all nodes on that path are also descendants of u,} \]
\[ \setminus \text{ so all w on that path } d[w] > d[u]; \]
\[ \therefore w \text{ is WHITE at time } d[w] \text{ (since } d[w] \text{ is later than } d[u]) \]

\[ \leftarrow \setminus \text{ Suppose } u \rightarrow v \text{ by WHITE path at time } d[u] \]
\[ \setminus \text{ but v doesn't become a descendant of u.} \]
WLOG, assume $v$ is 1st vert on path not a descendant of $u$. Consider vert $w$ that is immediately before $v$.

Then $d[u] \leq d[w] \leq f[w] \leq f[u]$ and

$$d[v] > d[u] \quad (v \text{ white at } d[u])$$
and $d[v] < f[w] \quad (b$ code$)$

$\therefore d[u] < d[v] < f[w] \leq f[u]$  

What can $f[v]$ be? either $wv$, $f[v] = f[u]$  

now $d[u] < d[v] < f[v] < f[u]$  

$\therefore v$ is a descendant of $u$
An edge \((u,v)\) if \(u\) is prior to \(v\)

Graph:

- 201 → 202 → 301 → 312 → 420 → 451
- 295
- 462

Ordering:

- Want valid ordering of classes
- i.e., ordering such that if \(u\) is before \(v\), no edge \((v,u)\)
- \(201\) \(202\) \(301\) \(420\) \(295\) \(302\) \(451\) \(462\)
- \(201\) \(295\) \(202\) \(301\) \(302\) \(462\) \(451\) \(420\)
- \(201\) \(295\) \(202\) \(301\) \(420\) \(302\) \(462\) \(451\)
Topological Sort: given DTG, find ordering s.t. 
\((u, v) \in E \text{ then } u \text{ is before } v\)

1) Do DFS; output verts in decreasing finishing time

\(O(V+E)\)

To show correctness, need \((u,v) \in E \text{ then } f[u] > f[v]\)

Consider color[v] at time \(d[u]\)

if WHITE then \(v\) is a descendant of \(u\)
    and hence \(f[v] < f[u]\)

GRAY then \(u\) is a descendant of \(v\)
    then \((u,v)\) is a back edge
    i.e. cycle

BLACK then \(v\) is already finished
    \(f[v] < d[u] < f[u]\)