strongly connected component
- maximal set of vertices that are all reachable from each other

SCC graph is a DAG
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Proof: Suppose cycle in SCC graph.
Pick 2 components $C, C'$ on cycle
Then 2 paths $C \rightarrow C'$ and $C' \rightarrow C$
Take any $u \in C$, $v \in C'$ can reach any other
in same SCC
Now $u \rightarrow v$ (go from $u$ to exit point of $C$ on
and $v \rightarrow u$. $C \rightarrow C'$, get to $C'$ then get
to $u$)
so $C, C'$ were not maximal $\Rightarrow$ $\Leftarrow$

Finding SCCs
1) do DFS, save $P$
2) compute $G^T$ ($G^T$ is $G$ w/ edges reversed)
3) DFS on $G^T$, visit verts in order of finishing time
(trees in $G^T$ forest are SCCs)
key facts: SCC graph is a DAG
SCCs of $G = SCCs$ of $G^T$

Thus $C, C'$ distinct SCCs in $G$ and $\exists (u,v) \in E$

$$f(C) > f(C')$$

( where $f(set) = \max f$ of vert in set
$= \min d$ of vert in set )

Proof Two cases 1) $d(C) < d(C')$

Let $x$ be 1st visited in $C$, consider graph at time $d(x) = d(C)$
Consider we C, at d[x] there is a white path X → w, so by WPT w is a descendant of x and so f[w] < f[x].

So f(C') < f[x] ≤ f(C)

2) d(C') < d(C)

At time f(C'), C is still white since no path C' → C (since is path C→C' and C'→C would give a cycle).

So f(C') < d(C) < f(C)

So by starting w/ last finishing SCC, we get just that SCC.

Let C be last finishing SCC.

If 3) finds C' from C then C'→C' is in C' and so C'→C in C
\[ f(c') > f(c) \]

contradicts \( C \) was last finishing.