Dijkstra's: no neg weight edges

d(v): weight of best path seen so far
\Pi(v): next-to-last vertex on that path
\text{Dijkstra}(s) \quad \text{using binary heap}

\begin{align*}
\forall v \in V: & \quad d[v] = \infty \\
\forall v \in V: & \quad \pi[v] = \text{NIL} \\
& \text{for all } u \in V \text{ in } \mathcal{O}(v) \text{ using key } d[v] \\
& \text{while } ( Q \neq \emptyset ) \\
& \quad v = Q.\text{extract-min()} \\
& \quad Q \rightarrow \text{for each } u \in \text{Adj}[v] \\
& \quad \quad \text{if } d[u] > w(v, u) + d[v] \quad \text{and } u \in Q \\
& \quad \quad \quad d[u] = w(v, u) + d[v] \\
& \quad \quad \quad \pi[u] = v \\
& \quad Q.\text{decrease-key}(u, d[u]) \\
& \text{in } \mathcal{O}(E \log V) \\
& \text{in } \mathcal{O}(E \log V)
\end{align*}
Invar: At $v$, $d[v]$ is correct ($d[v] = \delta(s, v)$) (cost of shortest path $s \to v$).

**Lemma:** Suppose $v_1 \to v_2 \to v_3 \to \ldots \to v_k$ is shortest path $v_i \to v_k$. Then $v_i \to v_{i+1} \to \ldots \to v_j$ is a shortest path $v_i \to v_j$ ($1 \leq i < j \leq k$); if it weren't, then $v_i \to \ldots \to v_j \to v_{j+1} \to \ldots \to v_k$ would be shorter than shortest $\Rightarrow$.

Suppose 1st mistake is at $v$. 
$d(v) = d(u) + w(u,v)$ since $\pi(v) = u$

$d(v) \leq d(y)$ since $v$ taken off at first

$d(v) \geq \delta(s,v)$ since $d(v)$ corresponds to some path

$d(x) = \delta(s,x)$ since $v$ is first mistake

$d(y) = d(x) + w(x,y) \leq \delta(s,y)$ since alg makes it

So when $x$ off $y$

$\delta(s,v) = \delta(s,x) + w(x,y) + \delta(y,v)$
\[ s(s, y) + s(y, v) \geq s(s, v) \quad \text{by no neg edges} \]

\[ = d[y] \]

\[ \geq d[v] \]

\[ s(s, v) \geq d[v] \]

\[ d[v] \geq s(s, v) \]

\[ \therefore s(s, v) = d[v] \quad \text{contradicts } v \text{ was a mistake} \]

**Bellman-Ford** (no neg cycles, can have neg edges)

for pass = 1 to \( V - 1 \)

for each edge \((u, v)\)

if \( d[v] > d[u] + w(u, v) \) then 2nd pass finds shortest paths using \( \leq k \) edges.

\[ d[v] = d[u] + w(u, v) \]

\[ \forall v \text{ such that } d[v] \text{ is} \]

\[ O(V \cdot E) \]

\[ 0 \rightarrow 0 \rightarrow 0 \]