Dijkstra from each vertex $(VE + V^2 \log V)$ using Fib-heap

Floyd-Warshall $O(V^3)$ better for sparse graphs

reweight edges using new fcn $\hat{w}$ s.t.

1) shortest path using w also shortest using $\hat{w}$

2) $\hat{w}(u,v) \geq 0$

can satisfy 1 w/ potential fcn $h(v)$, $\hat{w}(u,v) = w(u,v) + h(u) - h(v)$

consider path $p = v_0, v_1, v_2, \ldots, v_k$

$$\hat{w}(p) = \hat{w}(v_0,v_1) + \hat{w}(v_1,v_2) + \cdots + \hat{w}(v_{k-1}, v_k)$$

$$= w(v_0,v_1) + h(v_0) - \cancel{h(v_1)} + w(v_1,v_2) + \cancel{h(v_1)} - \cancel{h(v_2)} + \cdots$$

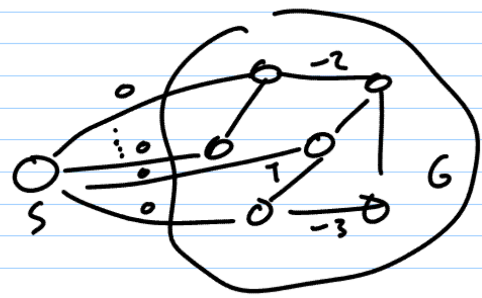
$$+ w(v_{k-1}, v_k) + \cancel{h(v_{k-1})} - h(v_k)$$

$$= w(p) + h(v_0) - h(v_k)$$

To find h to satisfy 2 as well:

Johnson

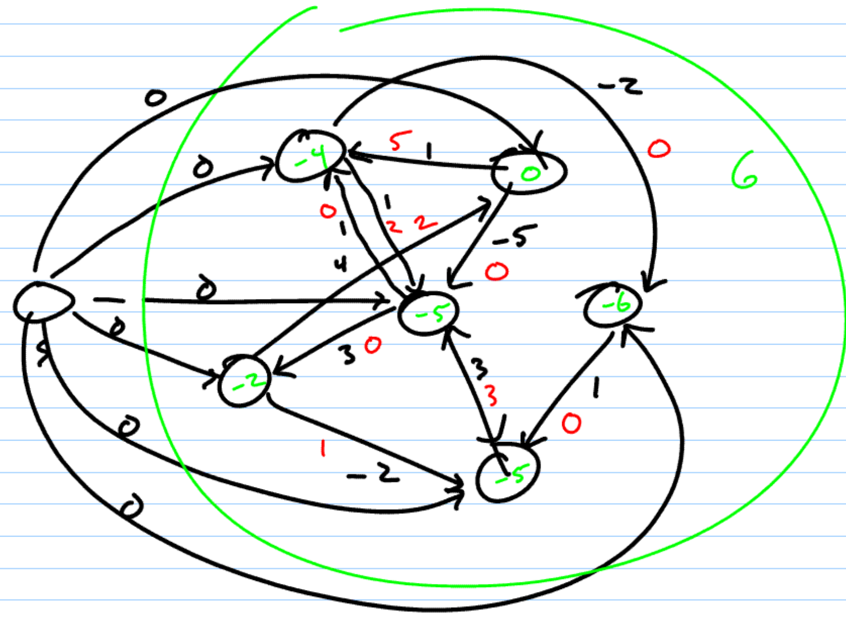
run B-F on G w/ new vertex s connected to all other verts using edge of weight 0



use s as source for B-F

B-F computes $\delta(s,v)$ for all $v \in G$

Set $h(v) = \delta(s,v)$



for any edge (u, v) $\quad \delta(s,v) \le \delta(s,u) + w(u,v)$

$$h(v) \le h(u) + w(u,v)$$

$$0 \le w(u,v) + h(u) - h(v) = \hat{w}(u,v)$$

factor (n)

```
int d=2
while (n>d)
    if n % d == 0
        output d
        n = n / d
    else
        d++
```

$O(n)$

not poly-time in size of input since size $= \log_2 n$

so $O(2^{size})$