antisymmetric: $\forall a, b,\ aRb \rightarrow b \not R a$

$\forall a, b,\ a \neq b \wedge (a,b) \in R \rightarrow (b,a) \notin R$

P = set of people

$R = \{(a,b) \mid b$ is an ancestor of $a\}$

R is antisymmetric

$\leq$ is antisymmetric (if $a \leq b$ and $a \neq b$ then $b \not\leq a$)

if R is reflexive, antisymmetric, transitive, then R is a partial order

$\leq$ is a partial order: $a \leq a$ • $a \leq b \wedge b \leq c \rightarrow a \leq c$

ancestor is too (assuming Jim is an ancestor of Jim)

a total order is a partial order such that $\forall a, b,\ (a,b) \in R$ or $(b,a) \in R$

$\leq$ is a total order

ancestor is not

Let $(a,b) \in R$ if $a = \sqrt[n]{b}$ for some $n \in \mathbb{Z}^+$

reflexive since $a = \sqrt[1]{a}$

antisymmetric: Suppose $a \neq b$ and $a = \sqrt[n]{b}$ for some $n \in \mathbb{Z}^+$

$a = (b)^{\frac{1}{n}}$

$a^n = b$

Suppose $b = \sqrt[m]{a}$ for some $m \in \mathbb{Z}^+$

then $a^{\frac{1}{m}} = b = a^n$

$a^{\frac{1}{m}} = a^n$ ← ok since $a \neq 1$

$n = \frac{1}{m}$

$\mathbb{Z}^+$ $\mathbb{Z}^+$

if $\frac{1}{m} \in \mathbb{Z}^+$ then $m = 1$ and $n = 1$

so $a = b$ $\Rightarrow\Leftarrow$

$\therefore$ there is no $m \in \mathbb{Z}^+$ s.t. $b = \sqrt[m]{a}$

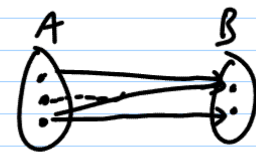
$\therefore$ $(b, a) \notin R$

transitive: if $a = \sqrt[n]{b}$ and $b = \sqrt[m]{c}$

then $a = \sqrt[n]{\sqrt[m]{c}} = \sqrt[n \cdot m]{c}$

so $(a, c) \in R$

Colin's relation is a partial order but not a total order $(2,3) \notin R$, $(3,2) \notin R$



a function f from A to B ($f: A \to B$) is a relation on A, B such that

$\forall a \in A, \underset{\text{exists exactly one}}{\exists!}\, b \in B$ s.t. $\underbrace{(a,b) \in R}_{f(a)=b}$

1-1 : $\forall a_1, a_2 \in A,\ f(a_1) = f(a_2) \to a_1 = a_2$

onto : $\forall b \in B,\ \exists a \in A$ s.t. $f(a) = B$

bijection: 1-1 and onto

Sets A, B are said to be equinumerous iff $\exists$ a bijection $f: A \to B$.

Ex: $\mathbb{Z}$ and $2\mathbb{Z}$ (even integers) are equinumerous (via the bijection $f(x) = 2x$)

finite : equinumerous with $\{1, \ldots, n\}$ for some n

infinite: not finite

countably infinite: equinumerous with $\mathbb{N}$

countable: finite or countably infinite

uncountable: not countable

To show A is countably infinite: list the elts in A

$$\begin{array}{ccccccccc} 2\mathbb{Z} = & 0, & -2, & 2, & -4, & 4, & -6, & 6, & \ldots\ldots\ldots \\ & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \end{array}$$

(implicity defines bijection $f(n) = \begin{cases} n & \text{if } n \text{ is even} \\ -(n+1) & \text{otherwise} \end{cases}$

If A is countable then $A \times A$ is countable
$A \times A \times A$ is countable
$\vdots$

If A is finite A^* is countable
So set of all programs is countable

$\{ f \mid f : \mathbb{N} \to \mathbb{N} \}$ is uncountable