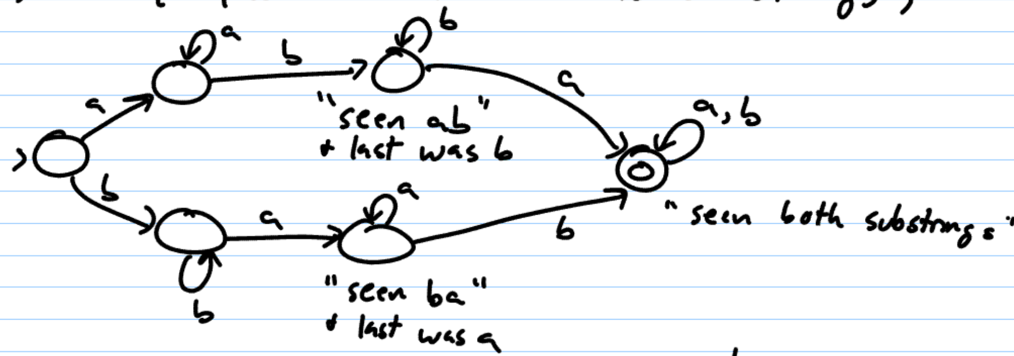
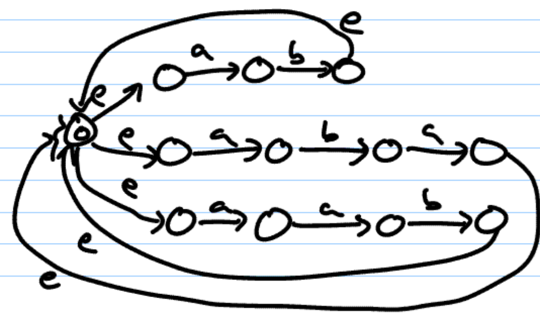
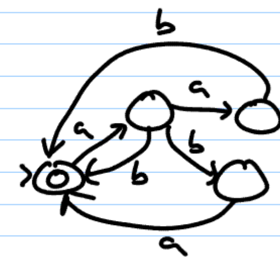
2.1.3 e) $\{w \mid w \text{ has } ab \text{ and } ba \text{ as substrings}\}$

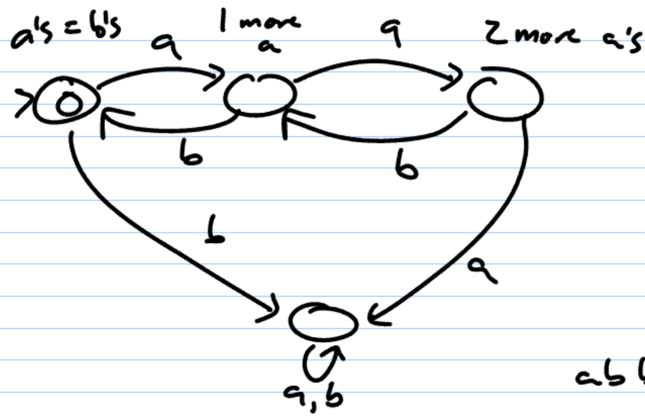


2.2.6a) $(ab \cup aab \cup aba)^*$





2.1.2



abba

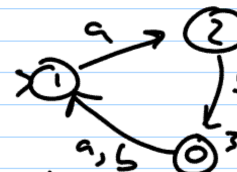
{w | w has equal # a's & b's and no prefix of w has more b's than a's or 3 more a's than b's}

$R(i,j,k)$ = regular exp for strings that drive M from q_i to q_j w/o using intermediate states higher than k

if start is q_s, final states are $q_{f_1}\ q_{f_2} \dots q_{f_k}$

$$R(s, f_1, n) \cup R(s, f_2, n) \cup \dots \cup R(s, f_k, n)$$

$$R(i,j,0) = \begin{cases} \displaystyle\bigcup_{\substack{a \in \Sigma \\ (q_i, a, q_j) \in \Delta}} a & \text{if } i \neq j \\ e \cup \left(\displaystyle\bigcup_{\Sigma} a \right) & \text{if } i = j \end{cases}$$



$R(1,2,0) = a$ $\quad R(1,1,0) = e$

$R(1,3,0) = \emptyset$ $\quad R(2,2,0) = e$

$R(2,3,0) = b$ $\quad R(3,3,0) = e$

$R(2,1,0) = \emptyset$

$R(3,1,0) = a \cup b$

$R(3,2,0) = \emptyset$

$$R(i,j,k) = R(i,j,k-1) \cup R(i,k,k-1)\,R(k,k,k-1)^*\,R(k,j,k-1)$$

Suppose w drives M from q_i to q_j using states $\leq k$

$$\overset{}{i}\ \overset{R(i,k,k-1)}{\text{------}}\ k\ \overset{R(k,j,k-1)}{\text{------}}\ j$$

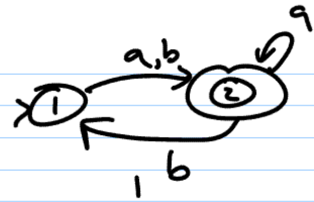
$$(q_i, w) \vdash \underbrace{(q_?, \ldots) \vdash \cdots \vdash (q_?, \ldots) \vdash}_{\text{nothing in here is } > k} (q_j, e)$$

if none are $= k$, then $R(i,j,k-1)$ already takes care of it

$$i\ \text{-----}\ k\ \text{-----}\ k\ \text{-----}\ j$$

$$R(i,k,k-1) \quad R(k,k,k-1) \quad R(k,j,k-1)$$

$$i\ \text{----}\ k\ \text{-----}\ k\ \text{------}\ k\ \text{-----}\ j$$



	$k=0$	$k=1$	$k=2$
$R(1,1,k)=$	ε	$\varepsilon \cup \varepsilon\varepsilon^*\varepsilon = \varepsilon$	$\varepsilon \cup (a\cup b)(\varepsilon\cup a\cup b(a\cup b))^* b$
$R(1,2,k)=$	$a\cup b$	$(a\cup b)\cup \varepsilon\varepsilon^*(a\cup b) = a\cup b$	$(a\cup b)\cup(a\cup b)(\varepsilon\cup a\cup b(a\cup b))^* \circ (\varepsilon\cup a\cup b(a\cup b))$ $= (a\cup b)(a\cup ba\cup bb)^*$
$R(2,1,k)=$	b	$b\cup b\varepsilon^*\varepsilon = b$	
$R(2,2,k)=$	$\varepsilon\cup a$	$(\varepsilon\cup a)\cup b\varepsilon^*(a\cup b) = \varepsilon\cup a\cup b(a\cup b)$	

$$\underset{i=1\ j=2\ k=1}{R(i,j,k)} = \underset{1,2,0}{R(i,j,k-1)} \cup \underset{1,1,0}{R(i,k,k-1)}\, \underset{1,1,0}{R(k,k,k-1)}^*\, \underset{1,2,0}{R(k,j,k-1)}$$

$i=1$ $j=1$ $k=1$ $1,1,0$ $1,1,0$ $1,1,0$ $1,1,0$