eliminate state 1

eliminate state 2
eliminate state 3

\[(\text{bua} \cup \text{b}) (\text{a} \cup \text{b})^* \text{b} \ (\text{a} \cup \text{b})^* \]

A language is regular \( \iff \) \( L \) is accepted by some NFA
\[ W = \varepsilon \text{ if } w = a^k b^k \text{ for some } k \geq 0 \] is not regular

**Pumping Theorem**

For all languages \( L \), if \( L \) is regular then there is some \( n \geq 1 \) such that if \( |w| \geq n \) and \( w \in L \) then \( w \) can be written as \( xyz \) where \( |xy| \leq n \), \( |y| > 0 \) and for all \( i \geq 0 \), \( xy^i z \in L \).

\[ W \] is not regular

**Proof:** Suppose \( W \) is regular. Then, by PT, there is some \( n \geq 1 \) s.t. \( \ldots \). Find that \( n \).

Then let \( w = a^n b^n \). \( |w| = 2n > n \) and \( w \in L \).

\( \therefore \) Can write \( w = xyz \) where \( |xy| \leq n \), \( |y| > 0 \) and
$\forall i \geq 0, \ xyiz \in L$.

If we write $w = xyz$ where $|xy| \leq n$, $|y| > 0$
then $x = a^p$, $y = a^q$ where $p \geq 0$ and $q \geq 0$.
But $x$ and $yz \in L$.
$xy = a^{p+q} b^n \notin L \text{ since } n+q \geq n$.

$\therefore W$ is not regular.

$B = \{ w \in \{a, b\}^* \mid w = a^k b b a^k \text{ for some } k \geq 0 \}$

$B$ is not regular.

Proof: Suppose it is. Find $n$ s.t. $\forall w, w \in L \implies |w| \leq 2n^2$.

Let $w = a^k b b a^k$. Then $w \in L \implies |w| = 2n + 2 \geq n$.

Write $w = xyz$ where $|xy| \leq n$ and $|y| > 0$.
Then $x = a^p$, $y = a^q$ where $p \geq n$ and $q > 0$.
Then $x \in L$ by $\text{---------}$ but $xz = a^{n+q} b b a^n \notin L \text{ since } n+q \geq n$.

So $xz \notin L \implies W$ is not regular.
$B$ is not regular.