2.4.3 (b) \( L = \{ w \mid w \text{ is decimal rep of a mult of } 7 \} \)

L is not regular.

"Proof": Let \( n \) be as required by PT.

[Goal: find \( w \) s.t. \( |w| \geq n \), \( w \in L \) but \( w \) can't be pumped no matter how you split it up]

Let \( w = 1.4.0^n \) so \( |w| = n+2 \geq n \) and \( w \in L \).

Problem: \( w \) can be pumped:

Let \( x = 1.4, y = 0 \) then \( |xy| \leq n \)

\( |y| > 0 \)

and \( xy^iz \in L \) for all \( i \geq 0 \)

Maybe we picked the wrong string.
Maybe $L$ is regular

Sum is 1 mod 3

115

47

0, 3, 6, 9

115 = 1 (mod 3)

36129

Adding trailing 4

13

1) multiply by 10
2) add 4

111 (mod 3)

0, 3, 6, 9

Sum is 2 mod 3

0, 3, 6, 9

Number read so far is 1 mod of 3

or

115 = 1 (mod 3)

115 - 4 = 2 (mod 3)

Number read so far is 2 more than multiple of 3

or

Sum is 2 mod 3

1, 4, 7

2, 5, 8

0, 3, 6, 9

Number read so far is a multiple of 3
add $x$ as last digit

1) mult by 10
2) add $x$
\[ L = \{ w w^R \mid w \in \{ a, b \}^* \} \]

\[ L \text{ is not regular} \]

By PT: Let \( n \) be as specified by PT.

[need to find \( w \in L, \| w \| \geq n \) that can't be pumped]

\[ a^n \text{ won't work} \]

Suppose \( w = x y z \) where \( |x y| \leq n \) and \( |y| > 0 \)

Then \( x = a^p, y = a^q, z = a^r \) where \( q < 0 \)

\[ x z = a^{n-q} b^n b^n a^n \notin L \]
Or use closure properties of regular languages:

- If \( L_1, L_2 \) regular so are
- \( L_1 \circ L_2 \)
- \( L_1 \cup L_2 \)
- \( L \cup L \)
- \( \Sigma^* - L \)
- \( L^* \)

Suppose \( L \) is regular. Then so is (by closure under \( \cap \))

\[ L \cap \{a^n b^n a^n \} \]

But \( L \cap \{a^n b^n a^n \} = \{a^n b^n a^n \} \)

where \( \{a^n b^n a^n \} \) is not regular

\[ \therefore L \text{ is not regular.} \]
Let $L = \{w \mid w \text{ has equal } # \text{ of } a's \text{ and } b's \}$

Suppose $L$ is regular.

Then so is $L \cap a^* b^* = a^n b^n$.

which is not regular $\Rightarrow$

$\therefore L$ is not regular

Let $L = \{w \mid w \text{ has 1 more } a \text{ than } b \}$

Suppose $L$ is regular.

Then $L \cap a^* b^* = a^{n+1} b^n$ is also regular.

(now use PT to show $a^{n+1} b^n$ not regular to get our $\Rightarrow$)

\[\text{known non-regular subset of } L\]
\[ L = \{ w | w \text{ has even } b' s \text{ and more } b' s \text{ than } a' s \} \]

\[ L \cap a^n b^k = \begin{cases} a^n b^n b^{2k} \mid n \text{ is even, } k \geq 0 \cup \{ a^n b^n b^{k+1} \mid n \text{ is odd, } k \geq 0 \} \end{cases} \]

\[ L' \]

\[ L' \text{ is not regular: find } n \text{ from PT} \]

(Could just use this w let \( w = a^n b^{2n+2} \) (so \( k \geq n \) and we \( L' \))

(L is not regular)

Write \( w = xyz \) when \( |y| \leq n \) and \( |y| > 0 \).

Then \( x = a^g \) \( y = a^h \) where \( g > 0 \).

Then \( |x| |y| |z| = a^n b^{2n+2} \)

\[ z = a^n b^{2n} b^2 \]

where \( n + |x| |y| |z| = n + |x| |y| |z| = n + \frac{2}{b} \geq 2n + 2 \)

so \( v \notin L' \Rightarrow \neg \]