1) \( L = \{ w \mid w \text{ has no } aaa \text{ and no } bbb \} \)

[2] different because \( e \cdot aaa \in L \) but \( a \cdot aa \notin L \)

[a] \( b \cdot bb \notin L \) but \( e \cdot bb, a \cdot bb \in L \)

[b] same as [2]

[aa] try a do distinguish

[bb] \( [aaa] = [bba] = [eacba] = [aaaa] \)
2) To show that $\exists$ infinite regular language

that's a subset of some

non-regular language

$a^*$ is infinite and regular

is $a^* \subseteq$ of some non-regular $L$

$a^* \not\subseteq a^*a^*$ no

$a^* \subseteq \{ww^R \mid w \in \{a,b\}^*\}$ $a^* \subseteq \{w \mid w=w^R\}$ Yes

not quite...

...but $(aa)^* \subseteq \{ww^R \mid w \in \{a,b\}^*\}$ $a^+ \subseteq \{w \mid \text{more a's than b's}\}$ Yes

$(ab)^+ \subseteq \{w \mid \text{same # of a's and b's}\}$
6) \( L = \{ w \mid w \text{ has } 2x \text{ as many a's as b's} \} \)

\[ a \rightarrow b \]

PT: \( L \) is regular \( \Rightarrow \exists n \text{ s.t.} \)

\[ \forall w, w \in L \land |w| \geq n \]

\[ \exists x, y, z \text{ s.t. } w = xyz \land |xy| \leq n \land n \geq 1 \]

\[ \forall i \quad x^i y^i z \notin L \]

To prove \( L \) not regular we prove \( \neg b \) and use

contrapositive to deduce \( \neg a \).

\[ \neg b : \quad \forall n \]

\[ \exists w \text{ s.t. } w \in L \land |w| > n \quad \]

\[ \forall x, y, z \mid w = xyz \land |xy| \leq n \land n \geq 1 \text{ then} \]

\[ \exists i \mid i \text{ s.t. } x^i y^i z \notin L \]
Imagine a game between you and PT. Proofs using PT show you have a winning strategy for this game—no matter what choices are made by PT, you can make a choice that leads to $xyizL$.

Suppose someone picks $n$.

We pick $w = a^i b^n$

No matter how someone splits $w$ into $xyz$ (subject to $|x| \leq n$, $|y| > 0$), we choose $i = 0$ so that $xy'z = xz \in L$

because $y = a^i$ and $xz = a^{2n-i} b^n$

where $2n - i < 2n$.
How do you determine if 2 DFAs accept the same language?

Case 1: same # of states in both languages
1) Find in machine 1 a string for every state
2) Check where those strings send you in machine 2
3) Verify that corresponding states in M1, M2 are same
   and equiv on every input, you go to equiv states

Case 2: diff # of states
1) minimize
2) check