\[ w \in \mathcal{W} \quad a^m b^n \quad S \rightarrow \varepsilon b \]

3.136) \[ w \in \mathcal{W} \quad a^m b^n \quad S \rightarrow \varepsilon \]

\[ S \rightarrow aSb \quad S \rightarrow \varepsilon \]
unclear
Pushdown automata: NFA with a stack

- Each transition may push something on the stack
- Consult stack to determine destination

For $a^nb^n$: read $a$ / push $a$ on stack
- Read $b$ / pop $a$ off stack

(retain states that keep track of which part - $a$'s or $b$'s - we're reading)

At least one way that $PDA$ accepts $w$:
1) $w$ drives $M$ to accepting state
2) With stack empty
Formally, PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$

- **States** $K$
- **Input Symbols** $\Sigma$
- **Stack Symbols** $\Gamma$
- **Start State** $s$
- **Final States** $F$

$\Delta$ is a relation on $K \times (\Sigma \cup \{\epsilon\}) \times \Gamma \times (K \times \Gamma^*)$

- Current state $K$
- Input symbol $(\Sigma \cup \{\epsilon\})$
- Stack symbol $\Gamma$
- Next state $K$
- New content on stack $(\Gamma \times \Gamma^*)$

**PDA for $a^n b^n$**:

- $\Delta$: $(q_0, a, e), (q_0, a)$, $(q_0, b, a), (q_1, e)$, $(q_1, b, a), (q_1, e)$

- $s = q_0$
- $F = \{q_0, q_1\}$
\( \text{aabb} \quad (q_0, aabb, c) \rightarrow (q_0, bb, a) \rightarrow (q_0, bb, aa) \rightarrow (q_1, c, a) \rightarrow (q_1, e, c) \quad q_1 \in F, \text{ stack is empty} \\
\quad \quad \text{so accept} \)

\( \text{a}^n\text{b}^n \quad - \text{when read } a, \text{ push } 2 \text{ a's on} \\
\quad \quad \text{when read } b, \text{ pop 1 a off} \\
\Delta = (q_0, a, e), (q_0, aa) \quad s = q_0 \)
\( (q_0, b, a), (q_1, e) \quad F = \{ q_0, q_1 \} \)
\( (q_1, b, a), (q_1, e) \)
\[ w \mid w \text{ has same # of } a's \text{ as } b's \]

\[ \text{stack keeps track of what you've seen more of} \]
\[ \text{baaabbabbaaa} \]
\[ \text{stack has } \text{bb} \]
\[ (s,e,e), (g_0,c) \]
\[ \text{bottom marker here seen } 2 \text{ extra } b's \]
\[ (g_0,a,a), (g_0,aa) \]
\[ \text{used to be more } a's, \text{ now even more excess } a's \]
\[ (g_0,b,b), (g_0,bb) \]
\[ (g_0,a,b), (g_0,e) \]
\[ \text{were more } b's, \text{ no fewer } \]
\[ (g_0,b,a), (g_0,e) \]
\[ (g_0,a,c), (g_0,a) \]
\[ \text{was balanced, now an extra } a \]
\[ (g_0,b,c), (g_0,b) \]
\[ (g_0,e,c), (f,e) \]
\[ \text{only final state} \]