P

big Jim ate (green cheese)

S

P V P

A P A P

big N green N

Jim cheese
\[a^n b^m c^p d^q \text{ where } n+m=p+q\]

\[
\begin{align*}
S & \rightarrow aScd & \text{aaabbccddd} \\
S & \rightarrow T | u | V & \text{aaabbcddd} \\
T & \rightarrow aTc \\
T & \rightarrow V \\
u & \rightarrow b Ud \\
u & \rightarrow V \\
V & \rightarrow bVc \\
V & \rightarrow e
\end{align*}
\]
PDA for $\{a^m b^m | n \leq m \leq 2n\}$

push a's while reading a's - push 1 or 2
pop a's while reading b's

aaa bbbbbb

can we do this deterministically?
3 definitions of PDAs

1) $w$ is accepted if end up in final state w/empty stack
   or get lost at any point
2) $w$ is accepted if end up w/empty stack
   (can put something on stack at beginning)
3) $w$ is accepted if end up in final state

all are equivalent

For example $1 \equiv 2$

Given $M$ of type 1, construct equiv machine of type 2
and given $M$ of type 2, construct equiv machine of type 1

Construct $M_2$ from $M_1$: 1) make up start stack symbol for $M_1$ say @

2) use same transitions, but we add $(f, e, @), (f, e)$ for
all final states \( f \) in \( M_1 \)

\[
\text{if } M_1 \text{ accepts } w, M_2 \text{ accepts } w
\]

(End up in \( M_1 \)'s final state with \( @ \) on stack,

new transition allows pop \( @ \), leaving empty stack,

so \( M_2 \) accepts)

also need to show if \( M_2 \) accepts \( w \) then \( M_1 \) accepts \( w \)

\[
\text{Construct } M_1 \text{ from } M_2
\]

Once we have construction, show

\[
M_1 \text{ accepts } w \iff M_2 \text{ accepts } w
\]
PDA \equiv CFG

1) Given CFG, construct equivalent PDA
2) Given PDA, construct equivalent CFG

1) idea: use stack to trace through derivation of string

\[ S \Rightarrow aSb \Rightarrow aaaSbb \Rightarrow aabbb \]

we will see (sort of) these on stack and then match w/ input string

for rule $S \Rightarrow aSb$, we add transition $(q_0, e, S) \rightarrow (q_0, aSb)$

do get $S$ in 1st place, have trans $(s, e, e) \rightarrow (q_0, SS)$

\[
\begin{align*}
(s, aabbb, e) & \rightarrow (q_0, aabbb, S) \\
& \rightarrow (q_0, aabbb, aSb) \\
& \rightarrow (q_0, aabbb, Sb)
\end{align*}
\]
also need \((q_0, o, \sigma^-) \rightarrow (q_0, e^-)\) \(\vdash (q_0, b, b)\) \(\vdash (q_0, e, e)\)