\[ H = \{ "M" "\langle w \rangle" \mid M \text{ halts on input } w \} \]

\[ H \text{ is not recursive. (No TM decides that set)} \]

\[ \text{It is impossible to create a program that detects infinite loops} \]

\[ H_1 = \{ "M" \mid M \text{ halts on } "M" \} \]

\[ H \text{ decidable} \rightarrow H_1 \text{ is decidable} \]

**Proof:** Suppose \( H \) is decidable via TM \( M_0 \).

Use \( M_0 \) to construct \( M_1 \) that decides \( H_1 \):

1. convert \( "M" \) to \( "M" "\langle M \rangle" \)
2. run \( M_0 \)
$H_1$ is not decidable.

Proof: Suppose $H_1$ is decidable. Then some TM $M_1$ decides $H_1$. Construct $M^*_w$ that:

1) input $w$
2) run $M_1$ on $w$
3) if $M_1$ says 'Y' $(M_1$ halts on $w$)
4) output $w$

$M^*_w$ semi-decides \( \overline{H_1} \) if $w$ is garbage or $w$ is encoding of $M$ and $M$ doesn't halt on $w$.

What does $M^*_w$ do on $"M^*_w"$?

Is $"M^*_w" \in \overline{H_1}$?

Suppose $"M^*_w" \in \overline{H_1}$. Then $M_1$ doesn't halt on $"M^*_w"$

But also $M^*_w$ halts on $"M^*_w"$ since $"M^*_w"$ is in the set $M^*_w$ semi-decides.

Suppose $"M^*_w" \notin \overline{H_1}$. Then $"M^*_w" \notin \overline{H_1}$ so $M^*_w$ halts on $"M^*_w$ within $k$ steps.

$M^*_w$ is not in $H_1$.
set that \(M^x\) semi-decides.

\[
\therefore H^x \text{ is not decidable}
\]

\[
\therefore H \text{ is not decidable.}
\]

**Given** \(M\), **does** \(M\) **halt on** \(e\)?

Undecidable! **Proof:** (we show that if we can solve this new problem we can also solve the halting problem)

\[\text{TM for halting problem}\]

**Input** \(M, w\)

Create \(M_w\) that:

1. **writes** \(w\) to tape
2. **does** what \(M\) does

\[\text{Ask} \ i \ \text{does} \ M_w \ \text{halt on} \ e?\]

\[\text{Return the answer}\]

**If** is possible to write, we’ve solved \(H\), can’t solve it, so must be impossible.