Given $M$, is there any $w$ s.t. $M$ halts on $w$?

A lg for determining if $M$ halts on $e$ already know undecidable.

reduced
“does $M$ halt on $e$” 1) Input $M$

“does $M$ halt on something” 2) Construct $\hat{M}^*$

3) Determine if there is some input $\hat{M}^*$ halts on

4) Output result of step 3

Construct $\hat{M}^*$ s.t. there is an input $\hat{M}^*$ halts on

if and only if

$M$ halts on $e$

$\hat{M}^*$ will

1) erase tape

2) do the same thing as $M$

(start with empty string as input)
NP: solvable in poly-time by some nondeterministic TM

$L_1 \leq_p L_2$ (\(L_1 \) polynomially reducible to \(L_2\))

means can solve \(L_1\) in poly time using \(L_2\) as subroutine

**Decision procedure for \(L_1\)**

1) Input \(w\) (we want to know if \(w \in L_1\) or not)
2) Construct \(w'\) construct such that
3) determine \(w' \in L_2\)
   \(w' \in L_2 \iff w \in L_1\)
4) output result of 3 and 
and copying \(w\) from \(w'\) takes polynomial time
Hamiltonian Path: given a graph \( G \), is there a path visiting each vertex exactly once?

Hamiltonian Cycle: given \( G \), is there a cycle that visits each vertex exactly once?

\[ HP \leq_p HC \]

**Alg for HP**

1) Input \( G \)

2) **Construct \( G' \) as follows**
   - add new vertex \( s \)
   - connect all \( v \in G \) to \( s \)

3) Determine if \( G' \) has HC

4) Output result of 3

need to make \( G' \) so \( G' \) has HC \( \iff \) \( G \) has HP

\( G \)
If $G$ has HP then $G'$ has HC
If $G'$ has HC then $G$ has HP

If $G$ has HP \[ u \rightarrow v \] then $G'$ has HC \[ s \rightarrow u \rightarrow v \rightarrow s \]
If $G'$ has HC \[ s \rightarrow u \rightarrow v \rightarrow s \] then $G$ has HP \[ u \rightarrow v \]

NP-hard: $L$ is NP-hard iff for all $L' \in \text{NP}$, $L' \leq_p L$

NP-complete: in NP and NP-hard

NP-complete problems exist!

Ex: CIRCUIT-SAT (given combinational circuit, can set input wires so output is on?)
If $A$ is NP-complete and $A \leq_p B$ then $B$ is also NP-complete.

If some NP-complete problem $A$ can be solved in poly time on standard TM, then anything solvable in poly-time on a nondeterministic TM can be solved in poly time on a standard TM.