Problem 0: Review old quizzes, homeworks, and exams.

Problem 1: Write negations of the following sentences.
(a) Rafael Palmerio uses steroids but he is not a good hitter.
(b) If Burt drives a Camaro then Candy likes Burt.
(c) Anyone who drives a BMW is rich.
(d) All dairy farmers have a machine that can milk any cow.

Problem 2: Show that the following argument form is valid.
\[ p \rightarrow q \\
\neg p \rightarrow r \\
\therefore q \lor r \]

Problem 3: Design a circuit with four inputs that outputs 1 if and only if at least three of its inputs are 1.

Problem 4: Let \( L(x, y) \) be the predicate “\( x \) lives in \( y \)”. Let \( S(x, y) \) be the predicate “\( x \) shops at \( y \)”. Let the domains of the variables and the truth of the predicates for particular values be as indicated by the tables given below. Use \( P \) for the set of people, \( C \) for the set of cities (ignoring the fact that Columbia and Timonium are not cities), and \( S \) for the set of stores.

<table>
<thead>
<tr>
<th></th>
<th>( L )</th>
<th>( S )</th>
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<tbody>
<tr>
<td></td>
<td>Columbia</td>
<td>Baltimore</td>
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<tr>
<td>Eastman</td>
<td>x</td>
<td></td>
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<tr>
<td>Glenn</td>
<td>x</td>
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<tr>
<td>Lawrie</td>
<td>x</td>
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<tr>
<td>Binkley</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Hall</td>
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Write each of the following English statements symbolically and determine whether they are true or false. Explain your answers briefly.
(a) Eastman shops at Giant.
(b) Everyone who lives in Columbia shops at Giant.
(c) No one shops at two different stores.
(d) There is a store whose patrons all live in Columbia.
(e) There is a place whose residents all shop at the same store.

**Problem 5:** Prove or disprove: for any integers \( a, b, c, \) and \( d, \) if \( a + b \mid d \) and \( a + c \mid d \) then \( b + c \mid d. \)

**Problem 6:** Find an integer \( r \) such that \( 0 \leq r < 13 \) and \( 11^8 \equiv r \pmod{13}. \)

**Problem 7:** Prove that \( \sqrt{6} \) is irrational.

**Problem 8:** Rewrite each of the following using summation notation.
(a) \( 4 + 7 + \cdots + 31 \)
(b) \( 8 + \cdots + n!n^2 \)
(c) \( -2 + 8 - \cdots + 8n^2 \)

**Problem 9:** Find a formula for \( \sum_{i=1}^{n} 4i + 3. \)

**Problem 10:** Find the smallest \( k \) such that any amount of at least \( k \) cents can be made with 4-cent and 5-cent stamps. Prove your answer.

**Problem 11:** Prove that for any sets \( A \) and \( B, \) \( A \cup (A \cap B) = A \) using no properties of sets other than the definitions of the set operations.

**Problem 12:** Prove that, for any sets \( A, B, C, \) and \( D, \) if \( C \subseteq A - B \) and \( D \subseteq B - A \) then \( C \) and \( D \) are disjoint.

**Problem 13:** Prove or disprove: for any sets \( A \) and \( B, \) \( \mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B). \)

**Problem 14:** A state issues license plates in two formats: either two letters followed by three numbers, or 5 numbers with a letter somewhere between them (but not at the ends). How many different license plates can the state issue?

**Problem 15:** Imagine a game like poker but played with 4 card hands. Which should be a better hand: one pair or “melting pot”, which is four cards all of different ranks and different suits.

**Problem 16:** 50 people were surveyed about what TV shows they watch. 21 reported that they watch The Amazing Race, 25 watch Veronica Mars, and 11 watch Arrested Development. 3 watch AD and TAR,
9 watch TAR and VM, 3 watch VM and 1 watches all 3. Fill in the Venn diagram that shows how many people watch each possible combination of shows.

**Problem 17**: 12 people are to be seated for a jury. The jury pool consists of 20 people: 7 college students, 10 retirees, and 3 working professionals. How many possible juries are there? How many have an equal number from each group? How many include no retirees? How many include at most 3 retirees? How many include all the college students? How many include more working professionals than college students?

**Problem 18**: The game Can’t Stop is played with 4 indistinguishable 6-sided dice. How many distinct outcomes are there of rolling the dice?

**Problem 19**: Define \( f : \mathbb{R} \to \mathbb{R} \) by \( f(x) = (x - 1)(x + 2)(x + 3) \). Is \( f \) 1-1? Is \( f \) onto? Explain your answers.

**Problem 20**: Prove that the set of squares of integers \( \{0, 1, 4, ...\} \) is countably infinite by finding a bijection between that set and \( \mathbb{Z}^+ \).

**Problem 21**: Prove that \( \mathbb{Z} \times \mathbb{Z} \) is countably infinite.

**Problem 22**: Is the set of infinite sequences of positive integers countable or uncountable? Justify your answer.

**Problem 23**: Order the following functions so that if \( f \) comes before \( g \), then \( f \) is of order at most \( g \) (\( f(x) \) is \( O(g(x)) \)). Indicate which functions can be switched in the lists (in other words, indicate which are of the same order as each other).

\[
x \log x \quad x \quad 2^x \quad 4^x \quad 8x + 1 \quad x^2 + x + 400 \quad x(\log x)^2
\]