poker hands w/ two pair

13	1) pick ~~one~~ higher rank	pick 4s	pick 6s
$\binom{4}{2}$	2) pick 2 cards of that rank	pick S D	H C
12	3) pick ~~other~~ lower rank	pick 6	pick 4s
$\binom{4}{2}$	4) pick 2 cards	pick H C	S D
44	5) pick last card	pick JH	pick JH

4S 4D 6H 6C JH

$$\frac{13 \cdot \binom{4}{2} \cdot 12 \cdot \binom{4}{2} \cdot 44}{2}$$

1) pick 2 ranks $\binom{13}{2}$
2) pick suits for higher $\binom{4}{2}$
3) suits for lower $\binom{4}{2}$
4) 5^{th} card 44

full house hands

1) pick 2 ranks $\binom{13}{2}$
2) pick rank for triple 2
3) pick suits for triple $\binom{4}{3}$
4) pick suits for double $\binom{4}{2}$

three of a kind hands

1) pick a rank 13
2) pick suits $\binom{4}{3}$
3) pick 2 more ranks $\binom{12}{2}$
4) pick suit for higher $\binom{4}{1}$
5) " lower $\binom{4}{1}$

Yahtzee: roll 5 6-sided dice $\qquad 6^5$ items $= 7776$

list all rolls: 11111, 11112, ... , 16666, 21111, ..., 26666, ..., 66666

listed 5 times ↑

11222 listed $\binom{5}{2} = 10$ times

make sure dice listed in ↑ order

11111, 11112, ..., 11116, 11122, ..., 16666, 22222, ... 66666

# of ways to choose r things from n

	order matters Y	n
repetition Y	n^r	$\boxed{}$ ← $\binom{n+r-1}{r}$
n	$P(n,r)$	$\binom{n}{r}$

list elts	11112	11334	23456
write # of each	410000	202100	011111
draw picture	....|.||||	..||..|.||	|.|.|.|.|.

↑ a string over {•, |} with 5 • and 5 |

↑ # dice $\qquad$ ↑ 6−1 partitions

↑ # sides

how many strings of 5 • and 5 | are there?

$$\binom{10}{5} = 252$$

r dice with n sides → r •, n−1 |

$$\binom{n+r-1}{r}$$

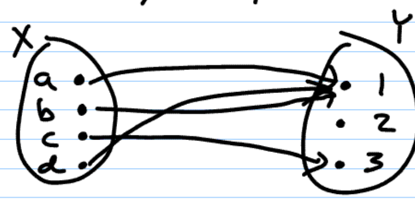
different outcomes of rolling $\underset{r}{3}$ $\underset{n}{10}$-sided dice?

$$\binom{12}{3}$$

$$P(\text{rolling } 11112) \neq \frac{1}{252}$$
$$= \frac{\#\text{ of rearrangements of } 11112}{\#\text{ outcomes}}$$
$$= \frac{5}{7776}$$

functions: a function f from X to Y $(f: \overset{\text{domain}}{X} \to \overset{\text{co-domain}}{Y})$ is a subset of $X \times Y$ s.t. for each $x \in X$ there is exactly one $y \in Y$ s.t. $(x, y) \in f$. (Each x is associated with exactly one y)

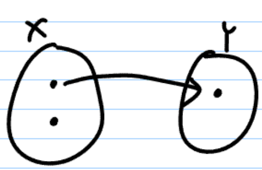
range = $\{1, 3\}$

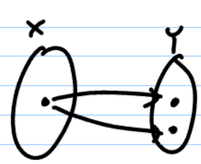


$f(a) = 1$
$f(b) = 1$
$f(c) = 3$
$f(d) = 1$

or

$\{(a,1), (b,1), (c,3), (d,1)\}$





$f: X \to Y$

range of f = $\{ y \in Y \mid \exists x \in X \text{ s.t. } f(x) = y \}$

inverse image of y = $f^{-1}(y) = \{ x \in X \mid f(x) = y \}$

$f^{-1}(1) = \{a, b, d\}$ $\quad f^{-1}(2) = \emptyset$ $\quad f^{-1}(3) = \{c\}$

identity fxn is fxn $i_X : X \to X$ defined by $i_X(x) = x$

let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 6x + 2$

let $g: \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Q}$ be defined by $g((x_1, x_2)) = \frac{x_1}{x_2}$

let $h: \mathbb{Q} \to \mathbb{Z} \times \mathbb{Z}$ be defined by $h(\frac{m}{n}) = (m, n)$

not well-defined $\quad h(\frac{2}{4}) = (2, 4)$

$h(\frac{1}{2}) = (1, 2)$

Let $k: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$ be $k(x_1, x_2) = \frac{x_1}{x_2}$

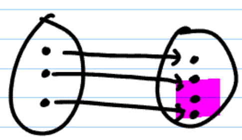
not well-defined

$f: X \to Y$ is said to be one-to-one (or injective) iff

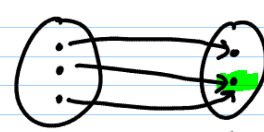
$\forall x_1, x_2 \in X, \; x_1 \neq x_2 \to f(x_1) \neq f(x_2)$

$\equiv$

$f(x_1) = f(x_2) \to x_1 = x_2$



one-to-one
not onto



not one-to-one
onto

Ex: Let $f: \mathbb{R} \to \mathbb{R}$ be def by $f(x) = 6x + 2$

f is 1-1

Proof: Assume $x_1, x_2 \in \mathbb{R}$ and $f(x_1) = f(x_2)$ [want $x_1 = x_2$]

Then $6x_1 + 2 = 6x_2 + 2$

so $6x_1 = 6x_2$

and $x_1 = x_2$

$\therefore$ f is 1-1

Ex: Let $f: \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Q}$ be defined by $f(x_1, x_2) = \frac{x_1}{x_2}$

f is not 1-1 counterexample: $f(2,4) = \frac{2}{4} = \frac{1}{2} = f(1,2)$

but $(2,4) \neq (1,2)$

Let $f: X \to Y$. f is said to be onto (or surjective) iff

$\forall y \in Y, \exists x \in X$ s.t. $f(x) = y$

$f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 6x+2$ is onto

Proof: Let $y \in \mathbb{R}$. [want $\exists x \in \mathbb{R}$ s.t. $f(x) = y$]

Let $x = \frac{y-2}{6}$. Then $x \in \mathbb{R}$. $\quad 6x+2 = y$

And $f(x) = f\left(\frac{y-2}{6}\right)$ $\quad x = \frac{y-2}{6}$

$= 6\left(\frac{y-2}{6}\right) + 2 = y$

same f but from $\mathbb{Z}$ to $\mathbb{Z}$ is not onto since nothing maps to 0

(would have to be $-\frac{1}{3} \notin \mathbb{Z}$)

$g: \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2$ is not onto

since $x^2 \neq -1$ for all $x \in \mathbb{R}$

$f: X \to Y$ is said to be a one-to-one correspondence (bijection) iff

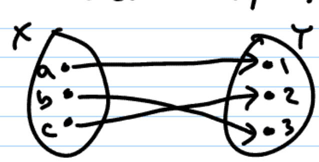
it is one-to-one and onto

Ex: $f(x) = 6x+2$ (as fcn $\mathbb{R} \to \mathbb{R}$) is a bijection

inverse fcns Let $f: X \to Y$ be a bijection

The inverse of f $f^{-1}: Y \to X$ is

defined by $f^{-1}(y) =$ the x s.t. $f(x) = y$



$f^{-1}(1) = a$

$f^{-1}(2) = c$

$f^{-1}(3) = b$

inverse of $f: \mathbb{R} \to \mathbb{R}$ def by $f(x) = 6x+2$ is

$$f^{-1}(y) = \frac{y-2}{6}$$

composition of fxns: If $f: X \to Y'$ and $g: Y \to Z$ and $Y' \subseteq Y$

then $\underline{g \circ f}: X \to Z$ is defined by $g \circ f(x) = \underline{g(f(x))}$

Let $f: \mathbb{R} \to \mathbb{R}$ be $f(x) = 6x+2$

and $g: \mathbb{R} \to \mathbb{R}$ be $g(x) = x^2$

then $g \circ f: \mathbb{R} \to \mathbb{R}$ is

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(6x+2) \\ &= (6x+2)^2 \end{aligned}$$

Suppose $f: X \to Y$ and $g: Y \to Z$ are both 1-1.

Then $g \circ f$ is also 1-1.

Proof: Suppose f, g are as given.

Let $x_1, x_2 \in X$ and suppose $g \circ f(x_1) = g \circ f(x_2)$

Then $g(f(x_1)) = g(f(x_2))$

$\therefore f(x_1) = f(x_2)$ since g is 1-1

$\therefore x_1 = x_2$ since f is 1-1

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$\subset$ means $\subseteq$