Lecture 10: Properties of Sets and Formal Methods

Theorem 5.2.1, Some Subset Relations

Inclusion of Intersection:

\[ A \cap B \subseteq A \text{ and } A \cap B \subseteq B \]

Inclusion in Union:

\[ A \subseteq A \cup B \text{ and } B \subseteq A \cup B \]

Transitivity of Subsets:

If \( A \subseteq B \) and \( B \subseteq C \), then \( A \subseteq C \)

To prove that a set \( X \) is a subset of another set \( Y \), prove that any element \( x \in X \) also belongs to \( Y \).

Theorem 5.2.2, Set Identities

The first 3 (Commutative, Associative, Distributive Laws) are familiar.

4. Intersection with U

5. Double Complement Law

6. Idempotent Laws \( A \cap A = A \cup A = \)

7. De Morgan's Laws (involve complements)
   \( (A \cup B)^c = (A \cap B)^c = \)

8. Union with U

9. Absorption Laws
   \( A \cup (A \cap B) = A \cap (A \cup B) = \)

10. Alternate Representation for Set Difference
    \( A - B = A \cap B^c \)
Proofs that 2 expressions involving sets are equal  Usually one of 2 types:

1. An element argument:

| To show that set \( X = \) set \( Y \), show that any element \( x \in X \) also belongs to \( Y \) AND any element \( x \in Y \) also belongs to \( X \). |

   So show the left side is a subset of the right side:
   
   \( i) \quad \text{Let } x \in \text{left side, show...} \)

   then show that the right side is a subset of the left side:
   
   \( ii) \quad \text{Let } x \in \text{right side, show...} \)

2. An argument from known set properties and already proven theorems—two column proof

\[
\begin{array}{ccc}
\text{proof} & = & \text{reason} \\
\text{} & = & \text{reason} \\
\text{} & = & \text{reason}
\end{array}
\]

To disprove a set equality, it is enough to show a counterexample. A Venn diagram (showing sets of points in the plane) can provide the counterexample.

Is a set statement true? First use a Venn diagram to get a sense of whether the statement is true or not. If true, use an argument. If false, provide a counterexample.

ex.  Show \(( A - B ) \cup ( A \cap B ) = A\)
ex. Show $A - (B - C) = (A - B) - C$ 

ex. Show $A - B \subseteq A$ 

ex. Prove if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. 