Lecture 11: More on Sets

To show that a set is empty, assume there is $x \in$ the set and reason to a contradiction.

ex. For all sets $A$ and $B$, prove $(A - B) \cap (A \cap B) = \phi$

Suppose $x$

ex. For all sets $A$, $B$, $C$ prove $(A - C) \cap (B - C) \cap (A - B) = \phi$
Most of what we have been doing for the last 4 weeks has involved **Boolean Algebras**.

A **Boolean algebra** is a set of objects for which **2 operations** are defined that obey the following:

<table>
<thead>
<tr>
<th>Boolean algebra (General)</th>
<th>Boolean Logic</th>
<th>Sets</th>
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<tbody>
<tr>
<td>+</td>
<td>∨</td>
<td>U</td>
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<tr>
<td>·</td>
<td>∧</td>
<td>∩</td>
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<tr>
<td>Commutativity of +</td>
<td>∨</td>
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<td>Comm. of ·</td>
<td>∧</td>
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<tr>
<td>Associativity of +</td>
<td>∨ ∨</td>
<td>U U</td>
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<tr>
<td>Assoc. of ·</td>
<td>∧ ∧</td>
<td>∩ ∩</td>
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<tr>
<td>Distrib. of + over ·</td>
<td>∨ ( ∧ )</td>
<td>U ( ∩ )</td>
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<td>Distrib. of · over +</td>
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<td>Identity for +</td>
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<td>Identity for ·</td>
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<tr>
<td>Existence of a Complement</td>
<td>p ∨ ~ p =</td>
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<td></td>
<td>p ∧ ~ p =</td>
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</table>
Properties of a Boolean Algebra.

Theorem 5.3.2 Properties of a Boolean Algebra

Let $B$ be any Boolean Algebra.

1. Uniqueness of the Complement Law: For all $a$ and $x$ in $B$, if $a + x = 1$ and $a \cdot x = 0$ then $x = \overline{a}$.

2. Uniqueness of 0 and 1: If there exists $x$ in $B$ such that $a + x = a$ for all $a$ in $B$, then $x = 0$, and if there exists $y$ in $B$ such that $a \cdot y = a$ for all $a$ in $B$, then $y = 1$.

3. Double Complement Law: For all $a \in B$, $(\overline{a}) = a$.

4. Idempotent Law: For all $a \in B$,
   \[(a) \ a + a = a \quad \text{and} \quad (b) \ a \cdot a = a.\]

5. Universal Bound Law: For all $a \in B$,
   \[(a) \ a + 1 = 1 \quad \text{and} \quad (b) \ a \cdot 0 = 0.\]

6. De Morgan's Laws: For all $a$ and $b \in B$,
   \[(a) \ \overline{a + b} = \overline{a} \cdot \overline{b} \quad \text{and} \quad (b) \ \overline{a \cdot b} = \overline{a} + \overline{b}.\]

7. Absorption Laws: For all $a$ and $b \in B$,
   \[(a) \ (a + b) \cdot a = a \quad \text{and} \quad (b) \ (a \cdot b) + a = a.\]

8. Complements of 0 and 1:
   \[(a) \ \overline{0} = 1 \quad \text{and} \quad (b) \ \overline{1} = 0.\]

Proofs in Boolean Algebra

Prove Idempotent: For all $a \in B$, $a + a = a$. 

Using Boolean algebra definitions and Idempotent, prove
For all $a$ and $b$ in $B$, $(a \cdot b) + a = a$. 