Lecture 13: Simple Probability

<table>
<thead>
<tr>
<th>Sample Space</th>
<th>Example</th>
<th>Enumerate Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>set of all possible outcomes of a random process</td>
<td>Flipping 2 coins</td>
<td>Getting exactly 1 tail</td>
</tr>
<tr>
<td>Event</td>
<td>a subset of the sample space</td>
<td></td>
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</tbody>
</table>

If S is a sample space with all outcomes *equally likely*, define the probability of event E,

\[
P(E) = \frac{\text{the number of outcomes in } E}{\text{the total number of outcomes in } S} = \frac{n(E)}{n(S)}
\]

ex. in flipping 2 coins, what’s the probability of getting exactly 1 tail?

ex. in flipping a coin what’s the probability of getting a head? (What's E?)

ex. in drawing a card, what’s the probability of drawing a face card?

Computing the Number of Possibilities When Order “Counts”

ex. How many ways different 4-letter strings can be formed with the alphabet \(a, b, c\)?

ex. How many 3-letter words can be formed?

ex. How many ways can the Orioles and the Yankees win or lose a 3 game series?
**Multiplication Rule:** If an operation consists of $k$ steps

<table>
<thead>
<tr>
<th>step $i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>can be performed</td>
<td>_____</td>
<td>_____</td>
<td>_____</td>
<td>. . .</td>
</tr>
<tr>
<td>in ways</td>
<td></td>
<td></td>
<td></td>
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</tbody>
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ex: How many user IDs are possible if each must consist of 5 characters and the first character must be an uppercase letter and the other 4 characters must be digits or uppercase letters?

Some problems are not so simple:

ex: How many results are there for the World Series?

<table>
<thead>
<tr>
<th>Game</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
</table>

**When Order Matters…..**

**Using Trees to Determine the Number of Possible Outcomes**

*(of a sequence of events)*

"*Draw the tree and count the leaves*"

**Example 1.** How many sequences of 22 flips of a coin exist?
**Example 2:** How many ways can you schedule your courses if CS can occur at 11 or 12, English at 10 or 2, and Physics at 10 or 1?

[See the club officer example, pp. 311-312]

**Example 3:** How many ways can a match in women’s tennis proceed to its end?
**Permutations**

If all $n$ distinct objects are rearranged in a sequence, this is called a permutation of the $n$ objects. The number of permutations of $n$ distinct elements is

ex. How many ways can the 15 kindergarten children form a line?

If only some, say $r$, of the $n$ distinct objects are arranged in an ordered sequence, this is called an $r$-permutation. The number of $r$-permutations of a set of $n$ distinct elements is

ex. How many ways can 5 of the 15 kindergarten children be selected to form a line?