

Lecture 14: More Probability

Counting the Number of Elements in a Set

(Remember, the elements in a set are NOT ordered.)

ex. How many words are possible of length 4 or less? (assume only lowercase letters are used)

Number of words of length 4 =

Number of words of length 3 =

ex. My grade report consists of 5 letter grades, one for each course, each an A, B, C, D, or F. How many possible grade reports will contain at least 1 A?

Number of possible reports =

Number of reports with at least 1 A =

Suppose all the grade reports are equally likely. What's the probability that I'll get all A's?

What is the probability of no A's?

ex. I roll a die 3 times. How many possible sequences of rolls will contain at least 1 one?

Number of possible sequences =

Number of sequences with at least 1 one =

What's the probability that I will roll at least 1 one?

- ex. In recent survey of 50 Loyola CS majors who can program in Java and/or C, 28 students said they can program only in Java and 12 said they can program only in C. How many can program in both?

Inclusion/Exclusion Rule for Sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

- ex. In recent survey of 70 Loyola CS majors who can program in Java and/or C and/or Prolog, 18 students said they can program in Java and C and 10 said they can program in C and Prolog, 8 said they can program in Java and Prolog. 46 claimed to be able to program in Java, 38 in C, and 16 in Prolog. How many can program in all 3 languages?

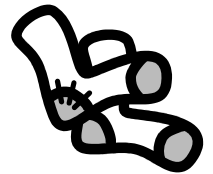
Combinations

ex: Choose 2 representatives from our class for as CS council. How many different combinations are there?

Does the order of the elements matter?

In these examples, underline the "key" word that indicates that order does NOT matter.

ex: How many pairs of socks are possible when selected from a drawer of 8 socks, each of a different color?



ex: How many groups of 3 letters can be selected from the set { a, b, c, d }?

ex: How many bridge foursomes are possible with members selected from this class?

An r-combination of a set of n elements is a subset of r of the n elements. Denoted

$$\binom{n}{r}$$

RELATIONSHIP BETWEEN PERMUTATIONS AND COMBINATIONS:

Number of k-combinations = $\frac{\text{Number of k-permutations}}{\text{The number of k-permutations containing the same set of elements}}$

Is the same as asking: "How many ways can a group of k elements be arranged in order?"

$$\binom{n}{k} =$$

Read as "

ex: How many groups of 6 students can I select from a group of 15 to ride in this car?

ex: How many a groups of 4 students can be formed from a class of 3 females and 5 males in which exactly one of the students is female are possible?

ex: Software Engineering teams of 4 members are being formed in a class of twenty students, of whom 6 are female and 14 are male. If all combinations are equally likely, what is the probability that a team will contain no females?

Will contain exactly 1 female?

Will contain at least 1 female?

Will contain at most 1 female?