Functions

A function is a mapping from one set, the *domain*, to another set, the *co-domain*.
Each element of the domain must have an image in the co-domain. We write **f: X→Y**.

domain:

co-domain:

range of *f*:

inverse image of 0, 1, 2:

Illustrating functions:

1. Arrow diagrams

2. Function machine

3. Using functional notation

4. Graphing

5. Listing all sets of ordered pairs

Two functions f and g are **equal** if they have the same domains and co-domains and for every x in the domain, f(x) = g(x).

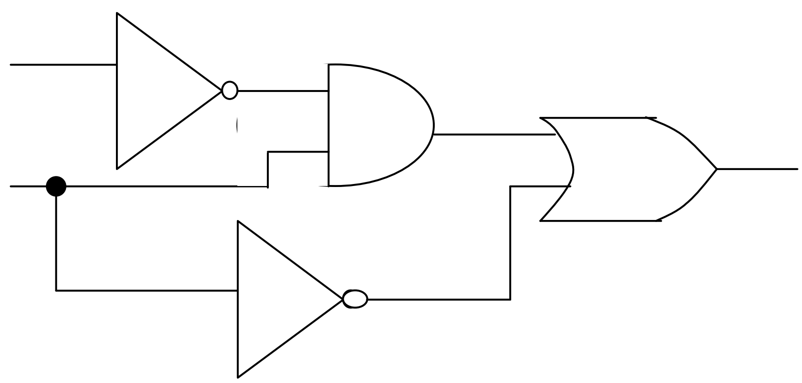
Examples of functions:

1. A **sequence** : 1, 1/2, 1/4, ... is a function from $\mathbf{Z}^+$ to **R**.

2. the **cardinality** of a subset is a function from $\mathcal{P}$(X) to non-negative integers.

   ex. X = { a, b, c } $\mathcal{P}$(X) =

3. a 2-input circuit is a function from $\{0,1\}\times\{0,1\}$ to $\{0,1\}$.



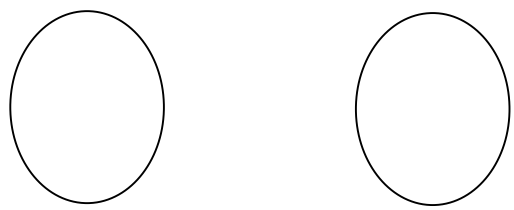
This circuit is equivalent to what statement?

This is an example of a **Boolean function**.

4. **Hamming distance** is a function from $\Sigma^n \times \Sigma^n$ to $Z^{nonnegative}$ where $\Sigma = \{ 0, 1 \}$

ex: $\Sigma^3 = \{$

One-to-one (Injective)



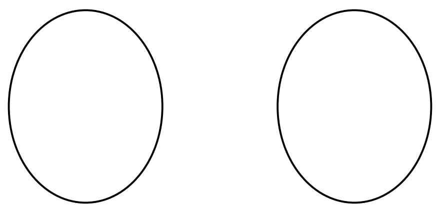
To show a function is injective, show that if $f(x_1) = f(x_2)$, then $x_1 = x_2$. **To show a function is NOT injective**, give a counterexample, i.e., show a case where $f(x_1) = f(x_2)$ but $x_1 \neq x_2$.

ex 1. Is the function *f*: **R** to **R** by *f*(x) = –5x + 1 is one-to-one? Show why.

ex 2. Is the function *g*: **R** to **R** by *g*(x) = x^2 + 2 is one-to-one? Show why.

ex 3. Is the function h: **R** to **R** by $h(x) = x^3$ is one-to-one? Show why.

Onto (Surjective) Every element in the co-domain is an image of some element in the domain.



To show a function is surjective, show that for every y ∈ co-domain, ∃ x ∈ domain ∋ f(x) = y.

To show a function is NOT injective, give a counterexample, i.e. find a y ∈co-domain ∋ there is no x ∈ domain ∋ f(x) = y.

ex. 1: Show whether the function f: **R** to **R** by $f(x) = x^2 + 2$ is onto.

ex. 2: Show whether the function *g*: **R** to **R** by *g*(x) = 3x + 1 is onto.

ex. 3: Show whether the function *h*: **R** to **R** by *h*(x) = x^3 is onto.

One-to-one and onto (Bijective or one-to-one corresponding)

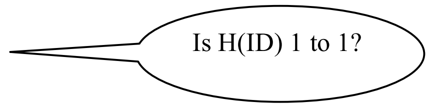
Is the function *f*: **R** to **R** by *f*(x) = 5x + 1 bijective?

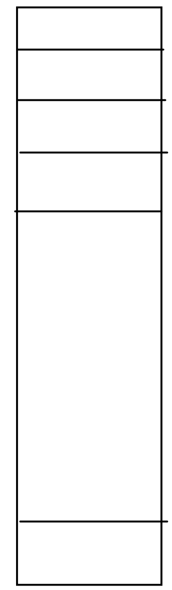
If f is one-to-one and onto, f has an inverse function, f^{-1}.

Hash functions - used to determine where to store data and where to find data that has been stored.

"Hash" the data by performing some mathematical operation on the key. The resulting number tells where to store the data.

ex. Store student records in an array indexed 0..999.
Hash the student ID as follows
$$H(ID) = ID \bmod 1000$$





What's the big advantage to a hash function?

Suppose we use the scheme

h(Student) = last 2 digits of the ID

as a hash function to store Student data in an array, i.e,

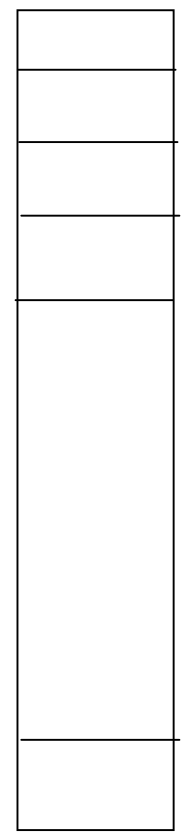
index = h(Student)

Where are Student records of students with IDs shown below stored?

091277817

112376149

116724516



Can there be a Problem?