Lecture 16: More on Functions

Pigeonhole Principle:
A function from one finite set to a smaller finite set cannot be one-to-one. There must be at least 2 elements in the domain that have the same image in the co-domain.

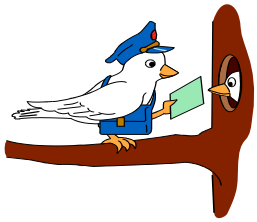
ex: In this room, must there be at least 2 people with the same residence area?

ex.: How many people would have to use the pigeonhole principle to guarantee that at least 2 people shared the same birth month?

Generalized Pigeonhole Principle:
For function f: X → Y where X and Y are finite sets and

$N(X) > k \cdot N(Y)$ *for some* $k \in \mathbb{Z}^+$,

then at least k + 1 elements of X have the same image under f.



16 people a room

ex: At least _____ people in the room have the same birth month.
Y has cardinality? Largest k = ?

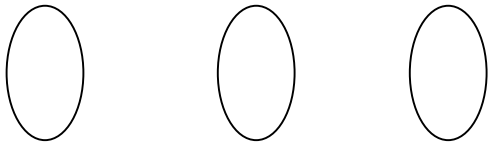
ex. At least _____ people in the room have the same sex.

ex. (Assume there are only 4 hair colors: blond, black, brunette, red hair)
At least _____ people in the room have the same hair color.

ex. (Assume there are only 3 majors: CS, MA, EG)
At least _____ people in the room have the same major.

Composition of Functions: Applying a function to the result of applying a function

$(g \circ f) (x) = g (f (x))$ Let g: **Z → Z** and f: **Z → Z**



ex. Let $f(x) = 2x$ and $g(x) = x^2 + 3$ Let g: **Z → Z** and f: **Z → Z**

Find $(g \circ f) (5)$

Find $(f \circ g) (5)$

Is there a function $h(x) = (f \circ g) (x)$? Is it 1 to 1? Onto?

Is there a function $j(x) = (g \circ f) (x)$?

If functions f and g are both one-to-one, $f \circ g$ is one-to-one.

If f and g are both onto, $f \circ g$ is onto.

If f and g are both one-to-one corresponding, $f \circ g$ is one-to-one corresponding.