Lecture 18: Finite State Automata

The wicked witch has hung her magic mirror and finds that, in spite of the owner's manual, the mirror can speak only 2 phrases:

"You're the fairest in the land."
"Snow White is looking better than you."

Suppose the mirror acquires another phrase:
"Snow White is history."

Finite State Automaton:

1. a set of states, S
2. a set of input symbols, I
3. a next-state **function** N (tells us where to go)

$$N : S \times I \rightarrow S$$

4. one state called the initial state s_0
5. one or more accepting states

If the FSA transition diagram involves ***no decision making***, it is a **deterministic FSA.**
(All transitions are "determined"—you have no choice.)

You display a (deterministic) FSA either
1. with a transition diagram or
2. with a (annotated) Next-State Table.

What is $N(s_0, a)$?
What is $N(s_2, b)$?

Input Symbols

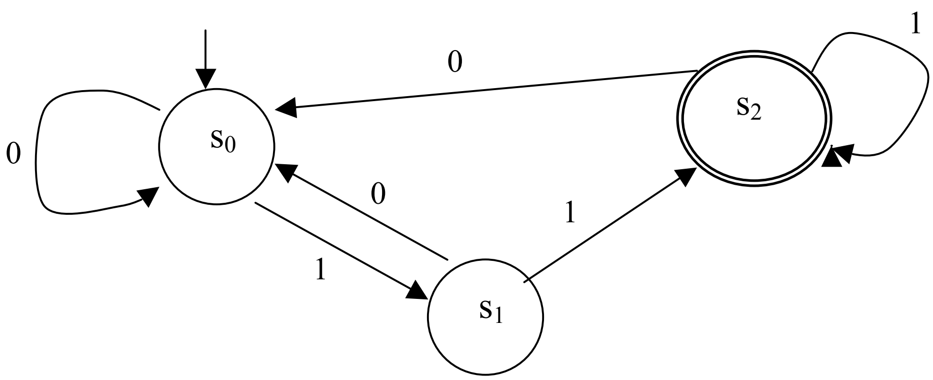
State	a	b
s_0		
s_1		
s_2		

Definition: The set of all strings accepted by a deterministic FSA is called a **regular language.**

Formally: $L(A) = \{ w \in I^* \mid N^* (s_0, w)$ is an accepting state and I is the alphabet $\}$
(N is the eventual state function, which is defined below.)*

Previously we referred to the alphabet as

Example 1. Consider the finite-state automaton *A*:



a) To what states does *A* go if symbols of the following strings are input to *A* in sequence, starting from the initial state?
   i. 11
   ii. 0101
   iii. 011011
   iv. 00110

b) Which of the strings in part (a) send *A* to an accepting state?

c) What is the language accepted by *A*?

d) Is there a regular expression that defines the same language?

Example 2. Draw a transition diagram for a FSA that accepts the language that consists of all strings that contain only a's and b's and end in b.

What are the input symbols (alphabet), I?

What is the annotated next-state table for this FSA?

Eventual state function N^* : $S \times I^* \rightarrow S$
Tells where where the string (in I^*) will eventually leave us.

For example 2: $N^*(s_0 , aba) =$ $N^*(s_1 , abaabb) =$

Example. 3. Draw a transition diagram for a FSA that accepts the language that consists of strings containing exactly 4 b's and $\Sigma = \{ a, b, c \}$.

$N^*(s_2 , abaacbcba) =$

$N^*(s_0 , cbaabbba) =$

Kleene's Theorem, Part 1
Given any language that is accepted by a finite state automaton, there is a regular expression that defines the same language.

Proof:

Suppose A is a finite-state automaton with a set I of input symbols, a set S of n states, and a next-state function N: $S \times I \rightarrow S$. Let I^* denote the set of all strings over I. Number the states s_1, s_2, s_3, …, s_n, using s_1, to denote the initial state, and for each integer $k = 1, 2, 3, \ldots, n$, let

$L_{i,j}^{k}$ = {$x \in I^*$ s.t. when the symbols of x are input to A in sequence, A goes from state s_i to s_j without traveling through an intermediate state s_h for which $h > k$}.

If s_j is an excepting state and if $k = n$ and $i = 1$,

Use mathematical induction to build up a set of regular expressions over I. Let the property $P(m)$ be the sentence, "For any pair of integers i and j with $1 \leq i, j \leq n$, there is a regular expression $r_{i,j}^{m}$ that defines $L_{i,j}^{m}$."

Show that the property is true for m = 0:

Show that for all integers k *with* $0 \leq k \leq n$*, if the property is true for* m=k *then it is true for* m=k+1*:*

}

Example 4. Draw a transition diagram for a FSA that accepts the language that consists of strings containing only a's and b's and consists of a number of a's followed by an equal number of b's.

<u>Can we use FSAs to represent what's going on in a program?</u>

```
public static void main()
{
        int i;
        int numE = 0;
        int numO = 0;
        Scanner scan = new Scanner (System.in);
        i = scan.nextInt ( );
        while ( i != 999)
         {    if (i % 2 == 0)
                   numE++;
              else
                   numO++;
              i = scan.nextInt ( );
         }
}
```