To prove two statements are logically equivalent:

1. 
   OR
2. 

Arguments

\[ \begin{align*}
&\text{premise}_1, \\
&\text{premise}_2, \\
&\vdots \\
&\text{premise}_n, \\
&\text{conclusion}
\end{align*} \]

ex. If you are a CS major, you will love this course.
   You are a CS major.
   \therefore You will love this course.

When we use p’s and q’s to represent a particular argument, this representation is called an argument form.

An argument form is valid if the conclusion is always true when

An argument is valid when its argument form is valid.

Here’s an argument you (may have) presented to your parents in high school. Is it valid?

If I have a rock concert on our deck, I am a cool.
I am cool.
\therefore I (should/will) have a rock concert on our deck.

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<th>p</th>
<th>q</th>
<th>p → q</th>
<th>q</th>
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If Kelly solved the problem, then Kelly is a genius or Kelly cheated.
Kelly did not cheat, and Kelly solved the problem.
\therefore Kelly is a genius.

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Two famous rules of inference:

Modus ponens: *(method of affirming)*

\[ p \rightarrow q \]

\[ p \]

\[ \therefore q \]

Modus tollens: *(method of denying)*

\[ p \rightarrow q \]

\[ \sim q \]

\[ \therefore \sim p \]

Is this argument valid?

*If you are a CS major, then you are held in awe by most Loyola students.*

*You are not held in awe by most Loyola students.*

*\therefore You are not a CS major.*

Is this argument valid?

*If you are a CS major or you are a math major, then you are held in awe by most Loyola students.*

*You are a math major or you are a CS major.*

*\therefore You are held in awe by most Loyola students.*

\[
\begin{array}{ccc}
p & q & r \\
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T & F & F \\
F & T & T \\
F & T & F \\
F & F & T \\
F & F & F \\
\end{array}
\]

Contradiction Rule:

\[ \sim p \rightarrow c \]

*If you are not tall, then the sky is blue and the sky is not blue.*

\[ \therefore p \]

\[
\begin{array}{cccc}
p & \sim p & c & \sim p \rightarrow c \\
\end{array}
\]

Proof by Contradiction

The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives who speak to you as follows:

*A* says: *B* is a knight.

*B* says: *A* and I are of opposite type.

What are *A* and *B*?

Suppose *A* is a knight.

\[
\begin{array}{cccc}
\end{array}
\]

Can a valid argument have a false conclusion?
**Fallacy** = error in reasoning that results in an invalid argument

**Converse error:** based on the assumption that the conclusion of the argument would follow from the premises if the premise $p \rightarrow q$ where replaced by its converse.

ex:   If you are a nerd, you are a CS major.  
      You are a CS major.  
      ∴ You are a nerd.

**Inverse Error:** based on the assumption

ex:   If you have a BMW, then you are rich.  
      You don’t have a BMW.  
      ∴

Validity and truth are different.  **Truth** depends on reality.  **Validity** depends on reasoning.

If an argument is valid, we are assured that the conclusion must be true whenever all the premises are true.

If I have two heads, I am a Martian.  
I have two heads.  
∴ I am a Martian.