Logic and Electricity

Every logical statement corresponds to an electrical circuit. Input lines to the circuit correspond to the simple statements involved.

Electrical power on = 1 = TRUE
Electrical power off = 0 = FALSE

Basic gates:

- NOT
- AND
- OR

A more complicated circuit. What is its corresponding logical statement?

Another:
Number Systems

**Base 10:** digits \{0, 1, 2, ..., 9\}

3789.13

**Base 2:** digits \{0, 1\}

1001101.1011

Converting from base 2 to base 10

Converting from base 10 to base 2 (recursive)

Algorithm:

**Step 1:** If the number is \( \geq 2 \), divide the number by 2; otherwise just write the number and stop.

**Step 2:** Perform the algorithm again using the quotient from step 1 as the number.

**Step 3:** Write the remainder from step 1.
Predicate Calculus

How do we represent the following with our logic symbols?

- All dogs have fleas.
- Some computer scientists are wacky.
- $x$ is a prime number.

A **predicate** is a statement containing a finite number of **variables**.

Symbolized $P(x, y, \ldots)$

The possible values for a variable is the **domain** of the variable.

Symbolized $D$.

The subset of $D$ in which are found values of $x$ for which predicate $P(x)$ is **true** is called the **truth set** of $P(x)$.

ex. $P(x)$: $x$ is a natural number and $x$ has no divisors except itself and one.

$$D = \{ x \in \mathbb{Z}^+ \mid x \text{ is a prime number} \}$$ (Recall $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{Q}$.)

<table>
<thead>
<tr>
<th>$P(x) \Rightarrow Q(x)$</th>
<th>means that the truth set of $P(x)$ is a subset of the truth set of $Q(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x) \Leftrightarrow Q(x)$</td>
<td>means that the truth set of $P(x)$ and the truth set of $Q(x)$ are the same</td>
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What is the relationship between $P(x)$ and $Q(x)$?

$P(x)$: $x$ is an integer ending in 0.
$Q(x)$: $x$ is an integer divisible by 5.

How can we change predicates into statements?

1. Replace the variable with a value.
   
   “$x$ is prime number” becomes “7 is a prime number”.

2. Explicitly state the domain.
   
   “$y$ is an even number” becomes “$y$ is an even number and $y \in \{2, 4, 6, \ldots\}$”
3. Use a quantifier:

<table>
<thead>
<tr>
<th>Quantifier</th>
<th>Symbol</th>
<th>Produces</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>for all</td>
<td>∀</td>
<td>universal statement</td>
<td>∀ x ∈ D, x is even</td>
</tr>
<tr>
<td>there exists</td>
<td>∃</td>
<td>existential statement</td>
<td>∃ x ∈ D  s.t. x is prime</td>
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</tbody>
</table>

**Informal English:**

Type of statement: Formal Quantified:

All dogs have fleas.

Some computer scientists are wacky.

Some programs produce incorrect output.

**Truth and Falsity of Statements**

<table>
<thead>
<tr>
<th>Quantifier</th>
<th>Prove it’s true</th>
<th>Prove it’s false</th>
</tr>
</thead>
<tbody>
<tr>
<td>∀</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∃</td>
<td></td>
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</tbody>
</table>

**Universal conditional statements**

∀ x ∈ D, if p(x), then q(x).

ex: ∀ s ∈ { students }, if s attends Loyola, s will receive a quality education.

*If the domain is restricted, the conditional can be removed.*

∀ s ∈ {

**Formalize in both ways:** (universal conditional & universal with restricted domain)

If a person has his/her tongue pierced, that person will have trouble eating spaghetti.