Minimum Spanning Tree

- Operates on graphs where we associate weights or costs with each edge
- Find the lowest-cost way to connect all of the points
- Often think of weights as distances
  - What is "the vertex closest to x"?
- Weights need not be proportional to distance
  - Time to accomplish a task
  - Represent the cost to traverse the edge

Euclidean MST

- Present problems in terms of distance to appeal to intuition
  - "short" edge ≈ "low-weight" edge
  - By inspection, longer edges have higher weights
  - Algorithms can take advantage of geometric nature

MST Definition

- A minimum spanning tree (MST) of a weighted graph
  - If weights are positive, MST is set of edges with minimal total weight that connects all vertices
  - If edges have equal weights, the spanning tree may not be unique

Nomenclature

- Creating minimal trees
  - one have the smallest weight
  - MST
    - minimal spanning tree
    - minimum-weight spanning tree
- Minimal – edge of minimum weight
- Maximal – edge of maximal weight

Representations

- Focus on weighted undirected graphs
- How do the dense and sparse graph representations have to be changed?
Parallel Edges

- Adjacency Matrix
  - No Parallel Edges
  - Policies for dealing with parallel edges
    - Keep the lowest weight
    - Keep the highest weight
    - Single edge is the sum of the weights

- Adjacency Lists
  - Keep parallel edges
  - Use one of policies mentioned above

Representing an MST

- Numerous options
  - A graph
  - A linked list of edges
  - A vector of pointers to edges
  - A vertex-indexed vector with parent links

- Choice is of little consequence
  - We can easily convert one representation to another